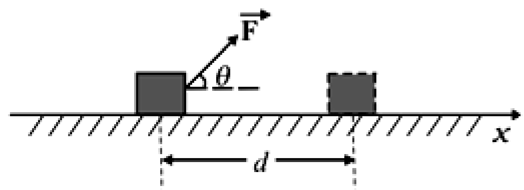
CHAPTER 5 : WORK, ENERGY AND POWER

WORK (W)

- The work done by the force is defined to be the product of component of the force in the direction of the displacement and the magnitude of this displacement, In other words the amount of workdone is defined as dot product of net force and displacement occurs in body.



$W = (F\cos\theta)\times S$ $\boxed{W = FS\cos\theta}$ $\boxed{W = \vec{F}\cdot\vec{S}}$

$\vec{F} =$ force by which workdone is being calculated

$\vec{S} =$ displacement in body

$\theta =$ Angle between force and displacement

- **No work** is done if the (i) displacement is zero,(ii) the force is zero,(ii) the force and displacement are mutually perpendicular to each other. e.g. ; In uniform circular motion workdone by centripetal force is zero as it is perpendicular to displacement.

UNITS OF WORK DONE

SI Unit : Kg- m²/sec² or Joule (J)

CGS unit : gm- cm²/sec² or erg $1\,\text{erg} = 10^{-7}$ joule

Other units : $1\text{eV} = 1.6\times10^{-19}$ joule $1\text{MeV} = 1.6\times10^{-12}$ J

1 kilo watt hour (KWH) = 3.6×10^{6} joule

SIGN OF WORK DONE

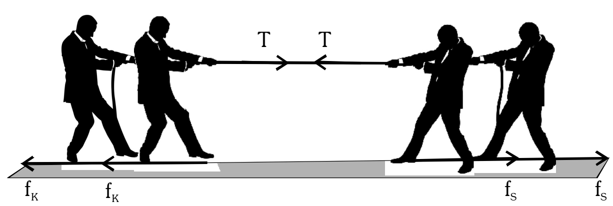
- Work done can be positive, negative or zero.

(a) If $\theta < 90^0$ (acute angle) $\cos\theta = +ve$ $W = +ve$

(b) If $\theta > 90^0$ (obtuse angle) $\cos\theta = -ve$ $W = -ve$

- If force is acting in the direction of displacement the workdone will be positive and if the force is acting just opposite to displacement workdone will be $-ve$.

Example : In a tug of war, the team that exerts a larger tangential force on the ground wins. Consider period in which a team is dragging opposite team by applying a larger tangential force on ground then



- Workdone by losing team on winning team through string is $W = -|Tds|$

$\leftarrow T$, $\rightarrow ds$ (displacement of point of contact)

- Workdone by ground on losing team is $W = -|f_k dl|$

  $\leftarrow f_k$, $\rightarrow$ dl (displacement of point of contact) therefore ,

- Workdone by ground on winning team is $W = 0$

  In this case displacement of point of contact of wining team is zero

- Workdone by ground on losing team is negative so total workdone by external force (by ground) on both team is negative.

CALCULATION OF WORK DONE

- **CASE I :** When force ($\vec{F}$) is constant

  $W = \vec{F} \cdot \vec{S}$ $\boxed{W = FS\cos\theta}$

  If $\vec{F} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$ $\vec{S} = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$ $\boxed{W = x_1x_2 + y_1y_2 + z_1z_2}$

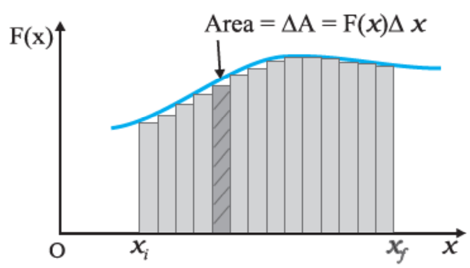
- **CASE II :** When force ($\vec{F}$) is variable

  $W = \int_{r_1}^{r_2} \vec{F} \cdot d\vec{r}$ $\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$ $d\vec{r} = dx\hat{i} + dy\hat{j} + dz\hat{k}$

  $W = \int_{(x_1,y_1,z_1)}^{(x_2,y_2,z_2)} (F_x\hat{i} + F_y\hat{j} + F_z\hat{k}) \cdot (dx\hat{i} + dy\hat{j} + dz\hat{k})$ $\boxed{W = \int_{x_1}^{x_2} F_x dx + \int_{y_1}^{y_2} F_y dy + \int_{z_1}^{z_2} F_z dz}$

- **CASE III :** By Force- Displacement graph

  Work done = area under graph (Take sign of area)



- **CASE IV :** By work - energy theorem

POINTS TO PONDER (PTP)

- Work is defined for an interval or displacement there is no term like instantaneous work similar to instantaneous velocity.
- For a particular displacement work is independent of time, work will be same for same displacement whether the time taken is small or large.
- When several forces acts, work by a force for a particular displacement is independent of other forces.
- Displacement depends on reference frame so work done by a force is reference frame dependent so work done by a force can be different in different reference frame.
- Effect of work is change in kinetic energy (K.E.)

CONSERVATIVE FORCE & NON-CONSERVATIVE FORCE

CONSERVATIVE FORCE

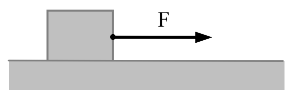
- A force is said to be conservative, if the work done, by or against the force is independent of path and depends only on initial and final positions i.e. does not depend on the nature of path followed between the initial and final positions.

- **Examples of conservative force:** All central forces are conservative like gravitational force, electrostatic force, electrostatic force, elastic force, restoring force due to spring etc.
- In presence of only conservative force total mechanical energy (K. E. +P. E.) of a body remains constant.
- The work done by a conservative force acting on a particle over a closed path is zero.

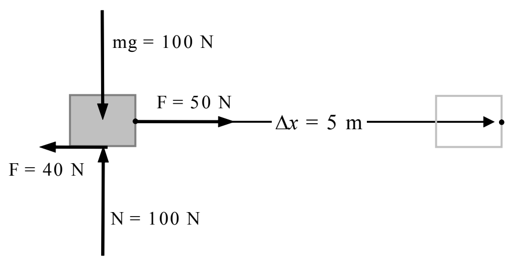
NON-CONSERVATIVE FORCE

- A force is said to be non-conservative, if work done, by or against the force in moving a body depends upon the path between the inital and final positions.
- The work done in a closed path is not zero in a non-conservative force field.
- The frictional forces are non-conservative forces. The work done against friction depends on the length of the path along which a body is moved. It does not depend only on the initial and final positions. The work done by frictional force is a round trip is not zero.

Example : A 10 kg block placed on a rough horizontal floor is being pulled by a constant force 50 N. Coefficient of kinetic friction between the block and the floor is 0.4. Find work done by each individual force acting on the block over displacement of 5 m?



Solution. Forces acting on the block are its weight (mg = 100 N), normal reaction (N = 100 N) from the ground, force of kinetic friction (f = 40 N) and the applied force (F = 50 N) and displacement of the block are shown in the given figure.



All these force are constant force, therefore we use equation $W_{i\to f} = \vec{F}\cdot\Delta\vec{r}$.

Work done W_g by the gravity i.e. weight of the block	$W_g = 0$ J	$(\because m\vec{g} \perp \Delta\vec{x})$
Work done W_N by the normal reaction	$W_N = 0$ J	$(\because \vec{N} \perp \Delta\vec{x})$
Work done W_F by the applied force	$W_F = 250$ J	$(\because \vec{F} \parallel \Delta\vec{x})$
Work done W_f by the force of kinetic friction	$W_f = -200$ J	$(\because \vec{f} \uparrow\downarrow \Delta\vec{x})$

Example : A particle is shifted form point (0, 0, 1 m) to (1 m, 1 m, 2 m), under simultaneous action of several forces. Two of the forces are $\vec{F}_1 = (2i + 3\hat{j} - \hat{k})$N and $\vec{F}_2 = (\hat{i} - 2\hat{j} + 2\hat{k})$N . Find work done by these two forces.

Solution; Work done by a constant force equals to dot product of the force and displacement vectors.

$$W = \vec{F}\cdot\Delta\vec{r} \qquad W = (\vec{F}_1 + \vec{F}_2)\cdot\Delta\vec{r}$$

Substituting given values, we have

$$W = (3\hat{i} + \hat{j} + \hat{k})\cdot(\hat{i} + \hat{j} + \hat{k}) = 3 + 1 + 1 = 5 \text{ J}$$

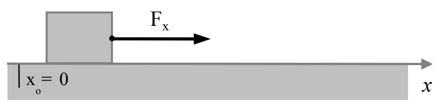
Example; Find the workdone by a force $\vec{F} = (-6x^3\hat{i})N$, in displacing a particle from x = 4 m to = –2m ?

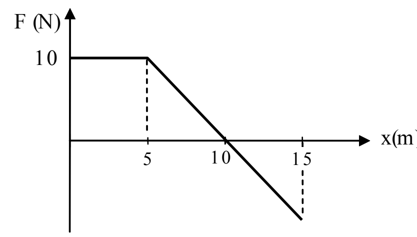
Solution ; $\vec{F} = (-6x^3\hat{i})N$,

$$W = \int_{x_1}^{x_2} F_x dx = \int_{4}^{-2} -6x^3 dx = \left[-6 \times \frac{x^4}{4}\right]_4^{-2} = \left[-\frac{3}{2}x^4\right]_4^{-2}$$

$$= \frac{-3}{2} \times 16 + \frac{3}{2} \times 256 = -24 + 384 = 360\,J$$

Example; A horizontal force F is used to pull a box placed on floor. Variation in the force with position coordinate x measured along the floor is shown in the graph.





(a) Calculate work done by the force in moving the box from x = 0 m to x = 10 m.

(b) Calculate work done by the force in moving the box from x = 10 m to x = 15 m.

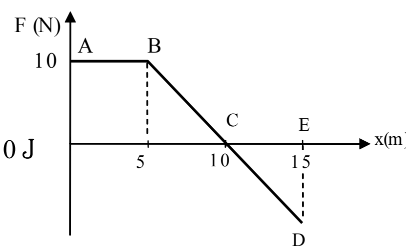
(c) Calculate work done by the force in moving the box from x = 0 m to x = 15 m.

Solution; In rectilinear motion work done by a force equals to area under the force-position graph and the position axis

(a) $W_{0\to10}$ = Area of trapazium OABC = 75 J

(b) $W_{10\to15}$ = –Area of triangle CDE = –25 J

(c) $W_{0\to15}$ = Area of trapazium OABC – Area of triangle CDE = 50 J



ENERGY (E)

- The energy of a body is defined as the capacity of doing work, in other words it is the factor or state of the body which makes the body capable to do work.
- Energy of a body gives us an idea of the total amount of work that the body can do. It has nothing to do with the time taken to do the work.
- It is a scalar quantity and the dimensional formula of energy is $[ML^2T^{-2}]$. It is the same as that of work.
- The unit of energy are the same as that of work i.e., joule in S.I. system and erg in CGS system.
- It exist in many forms like heat energy, sound energy, light energy etc. These forms of energy can change from one form to the other.
- Mass is the characteristic property of any material body by the virtue of which it exerts Gravitational Force on another body.
- The mass is variant i.e. the value of mass depends upon frame of reference, it's value increase on increasing the speed of body. The active mass is given as :

$$m = \frac{m_0}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$$

- **Mass energy equivalence:** Anything having mass has an equivalent amount of energy and vice versa, with these fundamental quantities directly relating to one another by Albert Einstein's famous formula.

$\boxed{E = mc^2}$ (mass is in kg) $\boxed{E = m \times 931 MeV}$ (mass is in amu)

- A consequence of the mass–energy equivalence is that if a body is stationary, it still has some internal or intrinsic energy, called its rest energy, corresponding to its rest mass.
- When the body is in motion, its total energy is greater than its rest energy, and equivalently its total mass (also called relativistic mass in this context) is greater than its rest mass.
- The rest mass is also called the intrinsic or invariant mass because it remains the same regardless of this motion, even for the extreme speeds or gravity considered in special and general relativity.

MECHANICAL ENERGY

- It is the macroscopic energy associated with **physical status** a system.In physical sciences, mechanical energy is the sum of potential energy and kinetic energy.

$$\boxed{E = K + U}$$

KINETIC ENERGY (K)

- Kinetic energy is the internal capacity of doing work of the object by virtue of its motion i.e. the K.E. of a body is the energy possesed by the body by virtue of its motion.
- This energy is associated with motion of the body 'OR' the energy of the body system due to state of motion.
- The motion of a body may be translational, rotational or combination of both. The kinetic energy associated with translational motion, is called as translational kinetic energy and the kinetic energy associated with rotational motion, is called as rotational kinetic energy.
- Quantatively the translational kinetic energy of the moving particle (over and above its energy at rest) is defined by $\boxed{K(v) = \frac{1}{2}mv^2 = \frac{1}{2}m\vec{v}.\vec{v}}$
- As mass m and $v^2(\vec{v}.\vec{v})$ are always positive, kinetic energy is always positive scalar, i.e, kinetic energy can never be negative.
- Kinetic energy depends on the frame of refernce e.g. kinetic energy of a person of mass m sitting in a train moving with speed v is zero with respect to frame of train but $\frac{1}{2}mv^2$ w.r.t. frame of reference of Earth.
- The kinetic energy is produced by conservative force, non conservative force and pseudo force.

RELATION BETWEEN MOMENTUM & KINETIC ENERGY

- Consider a body of mass m moving with velocity v. Linear momentum of the body, p= mv and the kinetic energy of a particle can be expressed as

$$K = \frac{1}{2}mv^2 = \frac{mv^2}{2} = \frac{p^2}{2m} \qquad \boxed{K = \frac{p^2}{2m}} \qquad \text{and} \qquad \boxed{p = \sqrt{2mK}}$$

- If a body has linear momentum, it implies that the body must have kinetic energy and vice versa.
- If a system of particle has linear momentum, it implies that the system has kinetic energy but it may be posible the system has kinetic energy and the linear momentum is zero(vector sum of linear momenta of all particles is zero).

WORK ENERGY THEOREM

- The time rate of change of kinetic energy is

$$\frac{dK}{dt} = \frac{d}{dt}\left(\frac{1}{2}mv^2\right) = m\frac{dv}{dt}v = F\,v \text{ (from Newton's Second Law)}$$

$$\frac{dK}{\cancel{dt}} = F\frac{dx}{\cancel{dt}}$$

Thus $dK = Fdx$

Integrating from the initial position (x_i) to final position (x_f), we have $\int_{K_i}^{K_f} dK = \int_{x_i}^{x_f} Fdx$

Where, K_i and K_f are the initial and final kinetic energies corresponding to x_i and x_f.

or $K_f - K_i = \int_{x_i}^{x_f} Fdx$ $\boxed{K_f - K_i = W}$

- From above it is clear that the work done by all the forces (conservative force, non-conservative force which may be internal or external, and pseudo force) on a body is equal to change in kinetic energy of that body.

$$\boxed{W_{CF} + W_{NCF} + W_{PF} + W_{external} = K_f - K_i}$$

- If there is no change in the speed of a particle, there is no change in kinetic energy. So work done by the resultant force is zero.
- If K.E. of the body decreases then work done is negative i.e. the force opposes the motion of the body.
- If K.E. of the body increases then work done is positive
- In above discussion we have assumed that the work done by the force is effective only in changing the kinetic energy of the body. It should however be remembered that work done on a body may also be stored in the body in the form of potential energy.

POTENTIAL ENERGY (U)

- The word potential suggests possibility of capacity for action. The term potential energybrings to one's mind 'stored' energy.
- The energy stored in a body or system by virtue of its configuration or its position is called potential energy.
- Regarding potential energy, it is worth nothing that :
  (i) Potential energy can be defined only for conservative forces. It does not exist for non-conservative forces.
  (ii) Potential energy can be positive, negative or zero.
  (iii) Potential energy depends on frame of reference.
  (iv) A moving body may or may not have potential energy.
  (v) Potential energy should be considered to be a property of the entire system, rather than assigning it to any specific particle.

- Change in potential energy is equal to negative of work done in shifting an object from some reference position to a given position for conservative force.

Therefore, $\Delta U = \int_i^f \vec{F}.\vec{dr}$ or $\boxed{U_f - U_i = \int_i^f \vec{F}.\vec{dr}}$

- Potential energy depends on position of reference level. It can be considered that at any reference level, F = 0 and potential energy U =0

- In case of intermolecular forces, for potential energy to be zero the reference level is at infinity.
- Potential energy depends on nature of force (i) For attractive forces, U is negative (ii) For repulsive forces, U is positive
- When work is done by conservative force is positive i.e. body moves in the direction of force, potential energy will decrease and if the work is done against the force (negative) i.e. is displaced opposite to the direction of force, potential energy will increase.
- If the workdone by conservative force is +ve then there will be loss in potential energy i.e. change in potential energy will be $-\text{ve}$ and vice versa.
- **Relation between force ($\vec{F}$) and potential energy (U) :-**

since $dU = -\int F dr \Rightarrow \boxed{F = -\frac{dU}{dr}}$

$$\boxed{\vec{F} = -\left[\frac{dU}{dx}\hat{i} + \frac{dU}{dy}\hat{j} + \frac{dU}{dz}\hat{k}\right]}$$

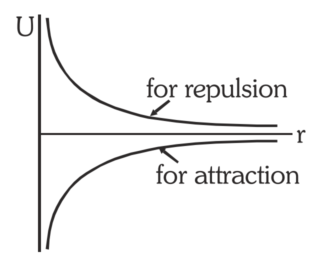
$\frac{dU}{dx}$ = Partial differentiation of U w.r.t. x (keeping y, z constant)

$\frac{dU}{dy}$ = Partial differentiation of U w.r.t. y (x, z constant)

$\frac{dU}{dz}$ = Partial differentiation of U w.r.t. z (x, y constant)

- **From U - r graph ;**

Slope of U-r graph $= \frac{dU}{dr}$ $F = -\frac{dU}{dr} = -$ (slope of U-r graph)



- If slope of U-r graph $\left(\frac{dU}{dr}\right) = +\text{ve}$ $F = -\text{ve}$ [Force is attractive]
- If slope of U-r graph $\left(\frac{dU}{dr}\right) = -\text{ve}$ $F = +\text{ve}$ [Force is repulsive]

Example: Given the potential energy of a body as U = xy +yz +zx joule. Find the expression of force on the body.

Solution: U = xy +yz +zx

$$F_x = \frac{-d}{dx}(xy + yz + zx) = -[y \times 1 + 0 + z \times 1] = -(y + z)$$

$$F_y = \frac{-d}{dy}(xy + yz + zx) = -[x \times 1 + z \times 1 + 0] = -(x + z)$$

$$F_z = \frac{-d}{dz}(xy + yz + zx) = -[0 + y \times 1 + x \times 1] = -(y + x)$$

$$\vec{F} = -(y + z)\hat{i} - (x + z)\hat{j} - (y + x)\hat{k}$$

GRAVITATIONAL POTENTIAL ENERGY

- The energy is due to workdone by gravitational force.

$$\boxed{U = -\frac{Gm_1m_2}{r}}$$

- It is always $-ve$, its maximum value is zero which is at infinite.
- While dealing the numerical, the potential energy of body can be considered zero at any level and the change in gravitational potential energy is given as

$$\boxed{\Delta U = mgh}$$ h = vertical difference between level of a body

- As the body moves away from the earth surface (gain height) potential energy increases and as it move towards the Earth surface potential energy decrease.

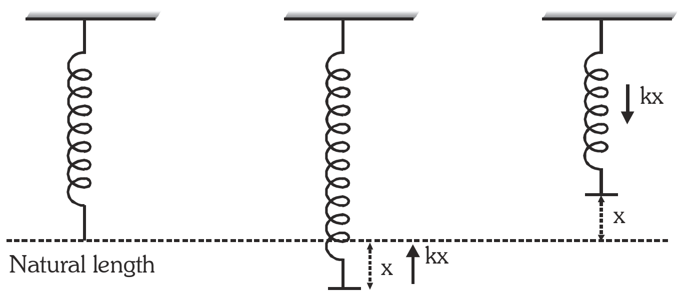
ELECTROSTATICS POTENTIAL ENERGY

- It is the potential energy due to electric force between two charge particles.

$$\boxed{U = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r}}$$

ELASTIC POTENTIAL ENERGY

- According to Hook's law, whenever an spring is stretched or compressed by a length 'x' from its natural length then restoring force- F = - kx [directed opposite to compression or expansion] is applied by spring due to its configuration.



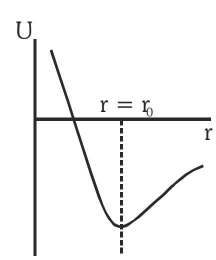
- Whenever spring is stretched or compressed, potential energy gets associated with it due to workdone against restoring force which is conservative in nature.
- If x is expansion/compression in spring then P.E. of spring (Energy stored in spring) is

$$\boxed{U = \frac{1}{2}kx^2}$$ k = force constant/ spring constant/ coefficient of stiffness

POTENTIAL ENERGY OF GASEOUS MOLECULE

- If reference is taken as infinity where potential energy is zero then the moleculesshas potential energy at a separation r due to intermolecular force.

$$\boxed{U = \frac{a}{r^{12}} - \frac{b}{r^6}}$$ a and b = constant.



- In the shown graph at $r = r_0$, P.E. = min. so particle is in stable equilibrium
- For $r < r_0$ Net Intermolecular force $\rightarrow$ repulsive

  $r > r_0$ Net Intermolecular force $\rightarrow$ attractive

  $r = r_0$ Net Intermolecular force $\rightarrow$ 0

POTENTIAL ENERGY & NATURE OF EQUILIBRIUM

- Equilibrium (mechanical equilibrium) is an state of body in which the net force as well as net torque on it is zero. This equilibrium has following two categories

  (a) Translational equilibrium $(\vec{F}_{net} = 0)$ (b) Rotational equilibrium $(\vec{\tau}_{net} = 0)$

- Different status of potential energy function in the state of equilibrium suggests us to define three different types of equilibriums – the stable, unstable and neutral equilibrium.

	Stable Equilibrium	Unstable Equilibrium	Neutral Equilibrium
Potential Energy (U)	Minimum	Maximum	Zero
$F=-\frac{dU}{dr}$	0	0	0
$\frac{d^2U}{dr^2}$	+ve greater than 0	–ve less than 0	0
$\frac{dF}{dr}$	–ve	+ve	0
Slope of U-r graph	0	0	0
Slope of F-r graph	–ve	+ve	0
When body is disturbed from equilibrium position	Returns back	Does not return back	Does not return back and acquire new equilibrium position

PRINCIPLE OF CONSERVATION OF MECHANICAL ENERGY

- From work energy theorem $W_{CF} + W_{NCF} + W_{PF} + W_{external} = K_f - K_i$ and $W_{CF} = -(U_f - U_i)$,

  we get $W_{NCF} + W_{PF} + W_{external} = (K_f + U_f) - (K_i + U_i)$

  $W_{NCF} + W_{PF} + W_{external} = E_f - E_i = \Delta E$ (Total Mechanical energy)

- If the internal forces are conservative but external forces also act on the system and they do work,

  $W_{NCF} = 0$ $W_{external} = E_f - E_i$

- If no external forces act (or the work done by them is zero) and the internal forces are conservative, the mechanical energy of the system remains constant, this is known as the principle of conservation of mechanical energy.
- If some of the internal forces are nonconservative, the mechanical energy of the system is not constant.
- If the internal forces are conservative, the work done by the external forces is equal to the change in total mechanical energy.

PRINCIPLE OF CONSERVATION OF ENERGY

- The total mechanical energy of the system is conserved if the forces doing work on it are conservative.
- If some of the forces involved are non-conservative, part of the mechanical energy may get transformed into other forms such as heat, light and sound but, the total energy of an isolated system does not change, as long as one accounts for all forms of energy.

- Energy may be transformed from one form to another but the total energy of an isolated system remains constant. Energy can neither be created, nor destroyed.
- Since the universe as a whole may be viewed as an isolated system, the total energy of the universe is constant. If one part of the universe loses energy, another part must gain an equal amount of energy.
- The principle of conservation of energy cannot be proved. However, no violation of this principle has been observed.
- The concept of conservation and transformation of energy into various forms links together various branches of physics, chemistry and life sciences.

POWER (P)

- The ratio of total work done to the total time, is called as average power and the rate of work done at any instant, is called as intantaneous power.

Average Power $\boxed{P = \frac{W}{\Delta t} = \frac{\text{change in Energy}}{\Delta t}}$

Intantaneous Power $\boxed{P = \frac{dW}{dt}}$

Also, $P = \frac{W}{\Delta t} = \frac{\vec{F} \cdot \vec{S}}{\Delta t}$ $\boxed{P = \vec{F} \cdot \vec{v}}$ $\boxed{P = F v \cos\theta}$ $\theta =$ angle between $\vec{F}$ & $\vec{v}$

- Calculation of power by Work - Time Graph :

$$P = \frac{dW}{dt} = \text{Slope of W-t graph}$$

- Also in case of Power - Time Graph :

$$P = \frac{dW}{dt} \Rightarrow \int dW = \int P dt \Rightarrow W = \int P dt$$

W = area under P - t graph

UNIT & DIMENSION OF POWER

- **SI unit** : J/sec or Watt
- **Other unit** : 1 KW = 1000 watt
  1 Horse Power (HP) = 746 watt
- **Dimension** : $P = \frac{W}{t} = \frac{ML^2T^{-2}}{T} = [ML^2T^{-3}]$

EFFICIENCY

- The energy or workdone given per second is called as input power and the energy or workdone obtained per second is called as output power.

$\boxed{\eta = \frac{\text{Output power}}{\text{Input power}}}$ $\boxed{\text{Percentage efficiency}(\%\eta) = \frac{[\text{Output power}]}{[\text{Input power}]} \times 100\%}$

EXAMPLE: A water pump light water a level 10 m below the ground. Water is pumped at the rate 30 kg/min with negligible velocity. Find the power of engine to do this.

mass per mint = 30 kg/min $= \frac{30}{60} = \frac{1}{2}$ kg / sec $\frac{m}{\Delta t}$ = mass / sec $= \frac{1}{2}$ kg / s

$$P = \frac{W}{\Delta t} = \frac{\text{change in PE of water}}{\Delta t} = \frac{mgh}{\Delta t} = \frac{1}{2} \times 10 \times 10 = 50\,W$$

EXAMPLE: A constant power P is applied to a car starting rest, then find the velocity and distance moved in terms of time t?

$P = F.v = mav = m \times \frac{dv}{dt} \times v$ $\quad \int vdv = \int \frac{P}{m} dt$ $\quad \frac{v^2}{2} = \frac{P}{m} t + c$ $\quad t = 0, v = 0$

$0 = \frac{P}{m} \times 0 + c$ $\quad c = 0$ $\quad \frac{v^2}{2} = \frac{P}{m} t$ $\quad v = \sqrt{\frac{2p}{m} t}$

$v \propto \sqrt{t}$ $\quad v \propto t^{1/2}$ $\quad v = \sqrt{\frac{2p}{m} t}$ $\quad \frac{dx}{dt} = \sqrt{\frac{2p}{m}} t^{1/2}$

$\int dx = \int \sqrt{\frac{2p}{m}} t^{1/2}.dt$ $\quad x = \sqrt{\frac{2p}{m}} \frac{t^{3/2}}{3/2}$ $\quad \boxed{x \propto t^{3/2}}$

COLLISION

- A collision is said to take place when either two bodies physically collide against each other or when the path of one body is changed by the influence of the other body.
- At the instant of collision the position vector of two bodies are same. Let the initial position vector of two bodies are $\vec{r}_1$ & $\vec{r}_2$, they are moving with velocities $\vec{v}_1$ & $\vec{v}_2$ and collide after time 't' at position $\vec{r}$.

  For first body - $\vec{r} = \vec{r}_1 + \vec{v}_1 t$ and for second body $\vec{r} = \vec{r}_2 + \vec{v}_2 t$

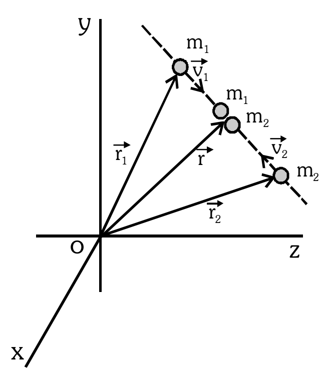
  $\vec{r}_1 + \vec{v}_1 t = \vec{r}_2 + \vec{v}_2 t$ $\qquad (\vec{v}_2 - \vec{v}_1)t = (\vec{r}_1 - \vec{r}_2)$

  Also $|\vec{v}_2 - \vec{v}_1| \times t = |\vec{r}_1 - \vec{r}_2|$

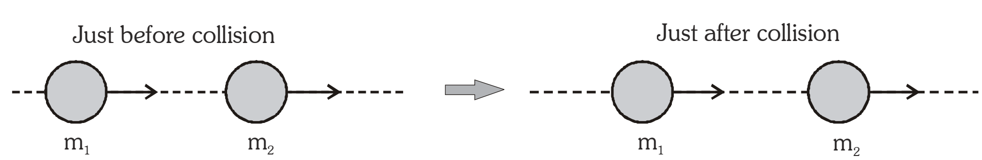
  $$t = \frac{|\vec{r}_1 - \vec{r}_2|}{|\vec{v}_2 - \vec{v}_1|}$$

  $$(\vec{v}_2 - \vec{v}_1) \times \frac{|\vec{r}_1 - \vec{r}_2|}{|\vec{v}_2 - \vec{v}_1|} = (\vec{r}_1 - \vec{r}_2)$$

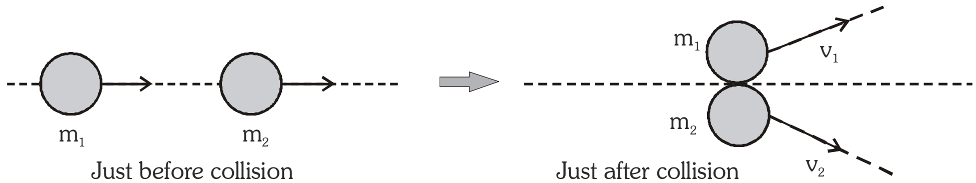
  $$\boxed{\frac{(\vec{v}_2 - \vec{v}_1)}{|\vec{v}_2 - \vec{v}_1|} = \frac{\vec{r}_1 - \vec{r}_2}{|\vec{r}_1 - \vec{r}_2|}}$$



- As a result of collision, the momentum and kinetic energy of the interacting bodies change.
- Forces involved in a collision are action-reaction forces, i.e., the internal forces of the system, other forces are not considered.
- In any type of collision since there is no external force during collision therefore linear momentum of system (vector sum of linear momenta of two bodies) remain conserved, total kinetic energy may or may not be conserved.
- On the basis of conservation of KE collision are of following three type -
  (1) Elastic Collision : Kinetic energy of system is conserved.
  (2) Inelastic Collision : Kinetic energy of system is not conserved.
  (3) Perfectly Inelastic Collision : Kinetic energy of system is not conserved.
- On the basis of line of motion just after collision, it is of following two types-
  (1) Head on or end on Collision : The two bodies move on the same line after collision as before collision.



(2) Non head on or Oblique Collision : The bodies move on different line of motion after collision with respect to before collision.



- In any type of collision motion of bodies is studied just after and just before collision.

HEAD ON ELASTIC COLLISION

- In this collision linear momentum as well as kinetic energy of system remains conserved but the linear momentum as well as kinetic energy of individual body may change.
- Let u_1 and u_2 are the velocity of m_1 and m_2 just before collision and v_1 and v_2 are the velocity just after collision.

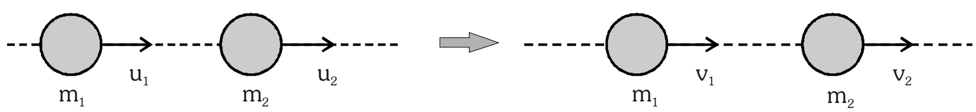
According to COLM : $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$ .... (i)

According to COKE : $\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$ .... (ii)

from equation (i) $m_1u_1 - m_1v_1 = m_2u_2 - m_2v_2$ .... (iii)

from equation (ii) $\frac{1}{2}m_1u_1^2 - \frac{1}{2}m_1v_1^2 = \frac{1}{2}m_2u_2^2 - \frac{1}{2}m_2v_2^2$ .... (iv)

from equation (iii) and (iv) $\frac{(u_1 - v_1)}{(u_1^2 - v_1^2)} = \frac{(u_2 - v_2)}{(v_2^2 - u_2^2)}$



$$\frac{(u_1 - v_1)}{(u_1 + v_1)(u_1 - v_1)} = \frac{(u_2 - v_2)}{(u_2 + v_2)(u_2 - v_2)}$$

$u_1 + v_1 = u_2 + v_2$ $\boxed{v_2 - v_1 = u_1 - u_2}$ $\boxed{v_2 - v_1 = -(u_2 - u_1)}$

- During elastic collision relative velocity of opprach just before collision is equal to relative velocity of receding just after collision

Again , $v_2 = v_1 + u_1 - v_2$

Putting value in equation 1

$m_1u_1 + m_2u_2 = m_1v_1 + m_2(v_1 + v_1 - u_2)$

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_1 + m_2v_1 - m_2u_2$

$v_1(m_1 + m_2) = (m_1 - m_2)u_1 + 2m_2u_2$

$\boxed{v_1 = \frac{(m_1 - m_2)u_1 + 2m_2u_2}{(m_1 + m_2)}}$ similary, $\boxed{v_2 = \frac{(m_2 - m_1)u_2 + 2m_1u_1}{(m_1 + m_2)}}$

- Put the sign of u_1 and u_2 in the formula of v_1 and v_2 consider the right side as +ve and left side as –ve direction.

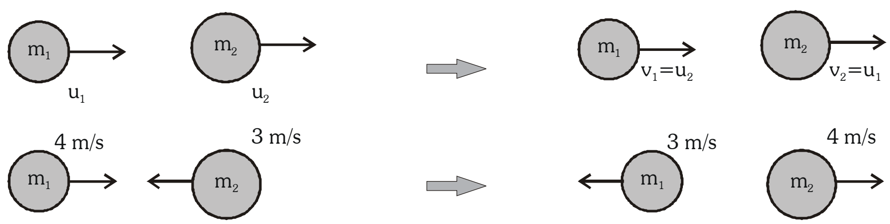
SOME SPECIAL SITUATIONS OF HEAD ON ELASTIC COLLISION

Case I : If $m_1 = m_2 = m(\text{say})$

$$v_1 = \frac{(m-m)u_1 + 2mu_2}{m+m} \qquad v_2 = \frac{(m-m)u_2 + 2mu_1}{m+m}$$

$$v_1 = u_2 \qquad v_2 = u_1$$

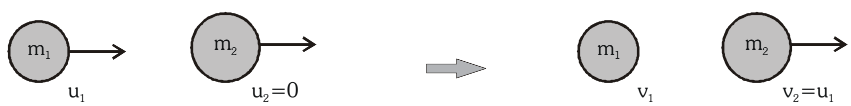
The two bodies will exchange their velocity magnitude and direction and maximum transfer of kinetic energy occurs.



Case II : If m_2 is at rest and $m_1 = m_2 = m(\text{say})$ $(u_2 = 0)$

$$v_1 = \frac{(m-m)u_1 + 2m\times 0}{m+m} \qquad v_2 = \frac{(m-m)u_2 + 2m\times u_1}{m+m}$$

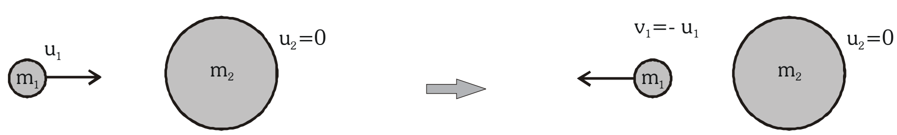
$$v_1 = 0 \qquad v_2 = u_1$$



Case III : If m_2 is at rest and $m_1 <<<<< m_2$ $[m_1 \longrightarrow 0]$ $[u_2 = 0]$

$$v_1 = \frac{(0-m_2)u_1 + 2m_2\times 0}{0+m_2} \qquad v_2 = \frac{(m_2-0)\times 0 + 2\times 0\times u_1}{0+m_2}$$

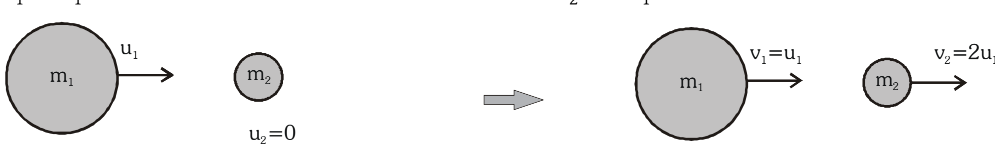
$$v_1 = -u_1 \qquad v_2 = 0$$



Case IV : If m_2 is at rest $(u_2=0)$ and $m_1 >>>> m_2$ $[m_2 \longrightarrow 0]$

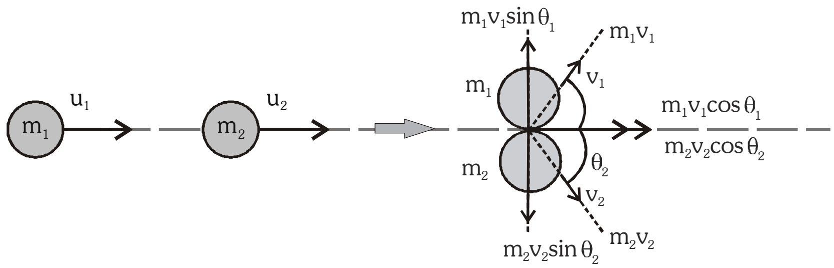
$$v_1 = \frac{(m_1-0)u_1 + 2\times 0\times 0}{m_1+0} \qquad v_2 = \frac{(0-m_1)\times 0 + 2\times m_1\times u_1}{m_1+0}$$

$$v_1 = u_1 \qquad v_2 = 2u_1$$



NON HEAD ON ELASTIC COLLISION OR OBLIQUE ELASTIC COLLISION

- Linear momentum as well as kinetic energy both are conserved.



$m_1v_1\sin\theta_1 = m_2v_2\sin\theta_2$

According to C.O.L.M. ,

$$m_1u_1 + m_2u_2 = m_1v_1\cos\theta_1 + m_2v_2\cos\theta_2 \qquad \text{........... (1)}$$

According to C.O.K.E. ,

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 \qquad \text{........... (2)}$$

COEFFICIENT OF RESTITUTION (*e*)

- It is the ratio of relative velocity of receding after collision and relative velocity of approach before collision.

$$\left[e = \frac{\text{Relative velocity of receding after collision}}{\text{Relative velocity of approach before collision}}\right] \qquad \boxed{e = \frac{v_2 - v_1}{u_1 - u_2}}$$

For elastic collision	For inelastic collision	For perfectly enelastic collision
$v_2 - v_1 = u_1 - u_2$	$v_2 - v_1 < u_1 - u_2$	$v_2 - v_1 = 0$
e = 1	**e < 1**	**e =0**

$$\boxed{0 \le e \le 1}$$

- The coefficient of restitution depends on nature of material of bodies which are colliding.

HEAD ON INELASTIC COLLISION

- In this collision linear momentum is conserved but kinetic energy is not conserved.
  According to C.O.L.M.

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \qquad \text{........... (i)}$$

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 \ne \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 \qquad \text{........... (ii)}$$

$$e = \frac{v_2 - v_1}{u_1 - u_2}$$

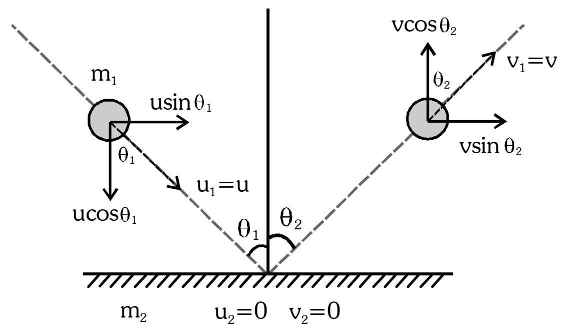
$v_2 - v_1 = eu_1 - eu_2$ $\qquad$ $v_2 = v_1 + eu_1 - eu_2$

Putting value in eq. 1 $\quad m_1u_1 + m_2u_2 = m_1v_1 + m_2(v_1 + eu_1 - eu_2)$

$$\boxed{v_1 = \frac{(m_1 - em_2)u_1 + (1+e)m_2u_2}{m_1 + m_2}} \qquad \boxed{v_2 = \frac{(m_2 - em_1)u_2 + (1+e)m_1u_1}{m_1 + m_2}}$$

Put the sign of u_1 and u_2 in above formula.

NON HEAD ON INELASTIC COLLISION OR OBLIQUE INELASTIC COLLISION



- For collision, only that component of velocity is responsible which is perpendicular to the surface.

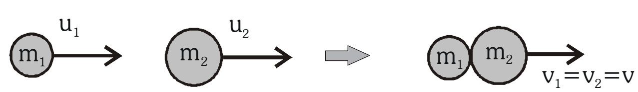
$$u\sin\theta_1 = v\sin\theta_2$$

$$e = \frac{v_2 - v_1}{u_1 - u_2} = \frac{0 - v\cos\theta_2}{u\cos\theta_1 - 0} \qquad \boxed{v\cos\theta_2 = -eu\cos\theta_1}$$

PERFECTLY INELASTIC COLLISION

- In this collision -
  - (a) Linear momentum of system is conserved.
  - (b) Maximum loss of kinetic energy occurs.
  - (c) Relative velocity after collision is zero i.e. **the two bodies got stuck during collision and move together just after collision.**

$$v_2 - v_1 = 0 \qquad \boxed{v_2 = v_1 = v\,(\text{say})}$$



- According to conservation of linear momentum

$$\vec{p}_1 + \vec{p}_2 = \vec{p} \qquad \vec{p}_1 + \vec{p}_2 = (m_1 + m_2)\vec{v}$$

$$m_1u_1 + m_2u_2 = (m_1 + m_2)v \qquad \boxed{v = \frac{m_1u_1 + m_2u_2}{(m_1 + m_2)}}$$

- Loss in kinetic energy $= K_i - K_f$

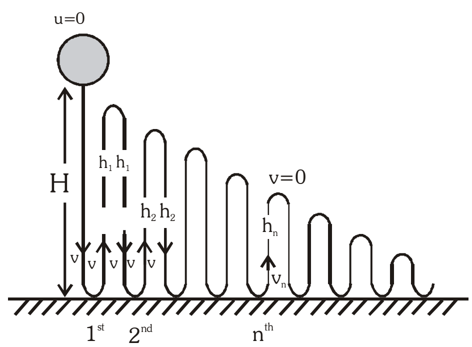
$$\Delta KE = \left(\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2\right) - \left[\frac{1}{2}(m_1 + m_2)v^2\right] \qquad \boxed{\Delta KE = \frac{1}{2}\left[\frac{m_1m_2(u_1 - u_2)^2}{m_1 + m_2}\right]}$$

- If $m_1 = m_2 = m$ (say)

$$\Delta KE = \frac{1}{2}\left[\frac{mm(u_1 - u_2)^2}{2(m + m)}\right] \qquad \boxed{\Delta KE = \frac{1}{4}m(u_1 - u_2)^2}$$

DISCUSSION

- Let a body be dropped from certain height H and it makes inelastic collision with the gr
collision). If h is the height attained by the body after nth collision then -



Before first collision $\quad v^2 = 0 + 2gH \qquad v = \sqrt{2gH}$

For first collision : $\quad u_1 = v, \quad u_2 = 0 \;, \quad v_1 = v, \quad v_2 = 0$

$$e = \frac{v_2 - v_1}{u_1 - u_2} = \frac{0 - v'}{v - 0} \qquad v' = ev$$

For second collision : $\quad u_1 = v', \quad u_2 = 0 \;, \quad v_1 = v'' \quad v_2 = 0$

$$e = \frac{0 - v''}{v' - 0} \qquad v'' = -ev' \qquad v'' = -e \times ev$$

$$v'' = -e^2 v$$

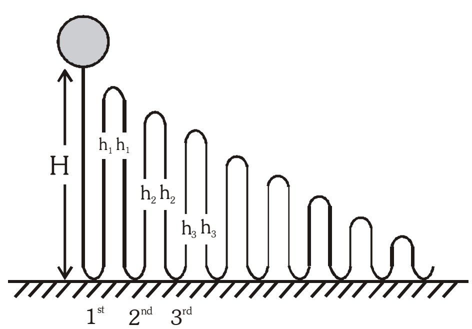
Therefore, after n^{th} collision : $\boxed{v_n = e^n v}$

After n^{th} collision : $\quad 0 = v_n^2 - 2gh \qquad v_n = \sqrt{2gh} \qquad v_n = e^n v$

$$\sqrt{2gh} = e^n \sqrt{2gH} \qquad \cancel{2g}h = e^{2n} \times \cancel{2g}H$$

$$\boxed{h = e^{2n}.H}$$

Total distance covered before stopping :



$$S = H + 2h_1 + 2h_2 + 2h_3 + \ldots\ldots\ldots\ldots\infty \qquad S = H + 2e^2H + 2e^4H + 2e^6H + \ldots\ldots\ldots\ldots\infty$$

$$S = H[1 + 2e^2(1 + e^2 + e^4 + ...\infty)]$$

$$S = H\left[1 + 2e^2\left(\frac{1}{1 - e^2}\right)\right]$$

$$S = H\left[\frac{1 - e^2 + 2e^2}{1 - e^2}\right]$$

$$\boxed{S = H\left[\frac{1 + e^2}{1 - e^2}\right]}$$

Total time taken before stopping:

$$H = 0 + \frac{1}{2}gt^2 \qquad t = \sqrt{\frac{2H}{g}} \qquad \text{similarly,} \qquad t_1 = \sqrt{\frac{2h_1}{g}}$$

$$T = t + 2t_1 + 2t_2 + 2t_3 + ...\infty$$

$$T = \sqrt{\frac{2H}{g}} + 2\sqrt{\frac{2h_1}{g}} + 2\sqrt{\frac{2h_2}{g}} + 2\sqrt{\frac{2h_3}{g}} + ...\infty$$

$$T = \sqrt{\frac{2H}{g}} + 2\sqrt{\frac{2e^2H}{g}} + 2\sqrt{\frac{2e^4H}{g}} + 2\sqrt{\frac{2e^6H}{g}} + ...\infty$$

$$T = \sqrt{\frac{2H}{g}}\left[1 + 2e + 2e^4 + 2e^3 + ...\infty\right]$$

$$T = \sqrt{\frac{2H}{g}}\left[1 + 2e(1 + e + e^2 + ...\infty)\right]$$

$$T = \sqrt{\frac{2H}{g}}\left[1 + 2e \times \frac{1}{1 - e}\right]$$

$$\boxed{T = \sqrt{\frac{2H}{g}}\left(\frac{1 + e}{1 - e}\right)}$$

NOTES :-