(iii) Trigonometric function of an angle $-\theta$ (negative angles)

$\sin(-\theta) = -\sin\theta$ $\cos(-\theta) = +\cos\theta$ $\tan(-\theta) = -\tan\theta$

(iv) On increasing $\theta$ from $0^0$ to $90^0$, the value of $\sin\theta$ and $\tan\theta$ increases.

(v) The value of $\cos\theta$ decreases when $\theta$ varies from $0^0$ to $180^0$.

(vi) The value of $\cos\theta$ lies between -1 & +1, $-1 \le \cos\theta \le +1$

(vii) If $\theta$ is small (say $< 5°$) then $\sin\theta \approx \theta$, $\cos\theta \approx 1$ and $\tan\theta \approx \theta$

## ADDITION/SUBTRACTION FORMULAE FOR TRIGONOMETRIC RATIOS

- $\sin(A+B) = \sin A\cos B + \cos A\sin B$
- $\sin(A-B) = \sin A\cos B - \cos A\sin B$
- $\cos(A+B) = \cos A\cos B - \sin A\sin B$
- $\cos(A-B) = \cos A\cos B + \sin A\sin B$
- $\sin 2\theta = 2\sin\theta\cos\theta$
- $\sin\theta = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$
- $\cos 2\theta = 2\cos^2\theta - 1 = 1 - \sin^2\theta = \cos^2\theta - \sin^2\theta$
- $\cos\theta = 2\cos^2\frac{\theta}{2} - 1 = 1 - \sin^2\frac{\theta}{2} = \cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2}$

# 2.3 CALCULUS

## FUNCTION

- A function describing a physical process expresses an unknown physical quantity in terms of one or more known physical quantities.
- We call the unknown physical quantity as dependent variable and the known physical quantities as independent variables.
- For the sake of simplicity, we consider a function that involves a dependant variable y and only one independent variable x. It is denoted y=f(x) and is read as y equals to f of x. Here f(x) is the value of y for a given x.
- Following are some examples of functions : $y = 2x+1$, $y = 2x^2+3x+1$, $y = \sin x$, $y = \ell n(2x+1)$
- Function of displacement (x) in terms of time (t): $x(t) = t^2+2t$
- Function of velocity (v) in terms of displacement (x) : $v(x) = x^2+3x+9$

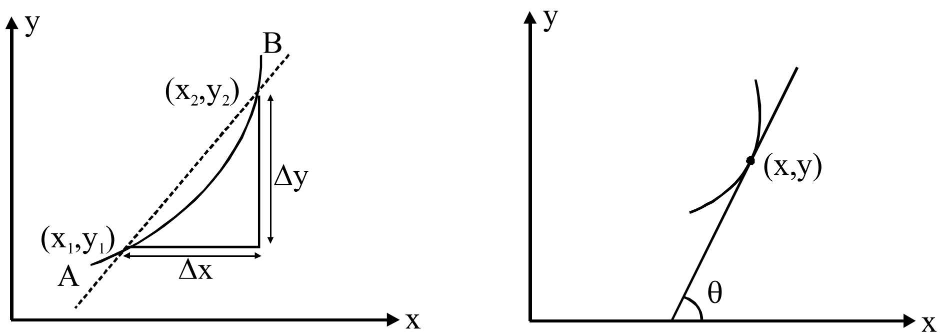
# 2.3.1 DIFFERENTIAL CALCULUS

- The purpose of differential calculus to study the change (i.e., increase or decrease) and the amount of variation in a quantity when another quantity (on which first quantity depends) varies independently.
- **Average rate of change :** Let a function $y = f(x)$ be plotted as shown in figure. Average rate of change in y w.r.t. x in interval $[x_1, x_2]$ is

$$\text{Average rate of change} = \frac{\text{change in y}}{\text{change in x}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \text{slope of chord AB}$$

- **Instantaneous rate of change :** It is defined as the rate of change in y with x at a particular value of x. It is measured graphically by the slope of the tangent drawn to the y-x graph at the point (x,y) and algebraically by the first derivative of function y =f(x).

Instantaneous rate of change $= \frac{dy}{dx}$ = slope of tangent = tan $\theta$




(a) (b)

- $\frac{dx}{dt}$ = Rate of change of displacement x w.r.t. time t = velocity
- $\frac{dv}{dt}$ = Rate of change of velocity v w.r.t. time t = Acceleration
- $\frac{dp}{dt}$ = Rate of change of linear momentum p w.r.t. time t = Force
- $\frac{d\theta}{dt}$ = Rate of change of angular displacement $\theta$ w.r.t. time t = Angular velocity
- $\frac{dW}{dt}$ = Rate of work done W w.r.t. time t = Power
- $\frac{dv}{dx} = \frac{\text{acceleration}}{\text{velocity}}$ ; $\frac{dv}{dx} \times \frac{dt}{dt} = \frac{\text{acceleration}}{\text{velocity}}$

## ● RULES FOR SIMPLE DIFFERENTIATION :

- f(x) = $x^n$ $\boxed{\frac{d}{dx}f(x) = \frac{d}{dx}x^n = nx^{n-1}}$

**Example:** (i) $\frac{d}{dx}x^3 = 3 \times x^{3-1} = 3x^2$ (ii) $\frac{d}{dx}x = 1 \times x^{1-1} = 1 \times x^0 = 1$

- f(x) = constant $\boxed{\frac{d}{dx}f(x) = \frac{d}{dx}\text{constant} = 0}$

**Example:** $\frac{d}{dx}(5) = 0$

- f(x) = constant $\times$ $x^n$ $\boxed{\frac{d}{dx}f(x) = \frac{d}{dx}\text{constt} \times x^n = \text{constt} \times nx^{n-1}}$

**Example:** $\frac{d}{dx}(5x^3) = 5 \times 3 \times x^{3-1} = 15x^2$

- f(x) = sin x $\boxed{\frac{d}{dx}\sin x = \cos x}$
- f(x) = sin ax $\boxed{\frac{d}{dx}\sin ax = a\cos ax}$

- $f(x) = \cos x$ $\quad \boxed{\frac{d}{dx}\cos x = -\sin x}$
- $f(x) = \cos ax$ $\quad \boxed{\frac{d}{dx}\cos ax = -a\sin ax}$
- $f(x) = \tan x$ $\quad \boxed{\frac{d}{dx}\tan x = \sec^2 x}$
- $f(x) = \tan ax$ $\quad \boxed{\frac{d}{dx}\tan ax = a\sec^2 ax}$
- $f(x) = e^x$ $\quad \boxed{\frac{d}{dx}e^x = e^x}$
- $f(x) = e^{ax}$ $\quad \boxed{\frac{d}{dx}e^{ax} = ae^{ax}}$
- $f(x) = \log_e x = \ln x$ $\quad \boxed{\frac{d}{dx}\log_e x = \frac{1}{x}}$
- $\boxed{\frac{d}{dx}\left[f(x) \pm g(x)\right] = \frac{d}{dx}f(x) \pm \frac{d}{dx}g(x)}$
- $\boxed{\frac{d}{dx}f(x)\times g(x) = g(x)\times\frac{d}{dx}f(x) + f(x)\times\frac{d}{dx}g(x)}$
- $\boxed{\frac{d}{dx}\frac{f(x)}{g(x)} = \frac{g(x)\times\frac{d}{dx}f(x) - f(x)\times\frac{d}{dx}g(x)}{\left[g(x)\right]^2}}$

**Example :** Find differentiation of y w.r.t x. (i) $y = x^2 - 6x$ (ii) $y = x^5 + 2e^x$ (iii) $y = 4\ln x + \cos x$

**Solution:** (i) $\frac{dy}{dx} = 2x^{2-1} - 6(1) = 2x - 6$ (ii) $\frac{dy}{dx} = 5x^{5-1} + 2e^x = 5x^4 + 2e^x$

(iii) $\frac{dy}{dx} = 4\left(\frac{1}{x}\right) + (-\sin x) = \frac{4}{x} - \sin x$

**Example :** Find first derivative of y w.r.t. x. (i) $y = x^2 \sin x$ (ii) $y = 4(e^x)\cos x$

**Solution:** (i) $\frac{dy}{dx} = \sin x \times \frac{d}{dx}x^2 + x^2 \times \frac{d}{dx}\sin x = \sin x \times 2x + x^2 \times \cos x = 2x\sin x + x^2\cos x$

(ii) $\frac{dy}{dx} = 4\left[\cos x \times \frac{d}{dx}e^x + e^x \times \frac{d}{dx}\cos x\right] = 4\left[\cos x \times e^x + e^x \times -\sin x\right] = 4e^x\left[\cos x - \sin x\right]$

**Example :** Find differentiation of y w.r.t. x. (i) $y = \frac{\sin x}{x}$ (ii) $y = \frac{4x^3}{e^x}$ (iii) $y = \frac{\sin x}{\cos x}$

**Solution:** (i) $f(x) = \sin x$, $g(x) = x$ $\quad \frac{dy}{dx} = \frac{x \times \frac{d}{dx}\sin x - \sin x \times \frac{d}{dx}x}{x^2} = \frac{x\cos x - \sin x}{x^2}$

(ii) $f(x) = 4x^3$, $g(x) = e^x$ $\quad \frac{dy}{dx} = \frac{e^x \times \frac{d}{dx}4x^3 - 4x^3 \times \frac{d}{dx}e^x}{\left(e^x\right)^2} = \frac{\cancel{e^x} \times 12x^2 - 4x^3 \times \cancel{e^x}}{e^{\cancel{2}x}} = \frac{12x^2 - 4x^3}{e^x}$

(iii) $\frac{dy}{dx} = \frac{d}{dx}\frac{\sin x}{\cos x} = \frac{\cos x \times \frac{d}{dx}\sin x - \sin x \times \frac{d}{dx}\cos x}{(\cos x)^2} = \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x$

## FUNCTION OF FUNCTION (CHAIN RULE)

$$\frac{d}{dx}f(y)=\frac{d}{dx}f(y)\times\frac{dy}{dy} \Rightarrow \boxed{\frac{d}{dx}f(y)=\frac{d}{dy}f(y)\times\frac{dy}{dx}}$$

**Example :** if $f(x)=(x^3+2x^2+5)^n$ $n\neq 1$

Let $x^3+2x^2+5=y$

$f(x)=y^n$ $\frac{d}{dx}y^n=\frac{d}{dy}y^n\times\frac{dy}{dx}$

$\frac{d}{dx}y^2=ny^{n-1}\times\frac{dy}{dx}$ $\frac{d}{dx}y^2=n(x^2+2x^2+5)^{n-1}\times\frac{d}{dx}(x^3+2x^2+5)$

$$\frac{d}{dx}y^2=n(x^2+2x^2+5)^{n-1}\times(3x^3+4x)$$

**Example :** $f(x)=\sin x^2$ $\frac{d}{dx}f(x)=\frac{d}{dx}\sin x^2=2x\times\cos x^2=2x\cos x^2$

**Example :** $f(x)=\cos\,\omega t$ $\frac{d}{dx}f(x)=\frac{d}{dt}\cos\omega t=\omega\times\sin\omega t=-\omega\sin\omega t$

**Example :** $f(x)=e^{x^2}$ $\frac{d}{dx}e^{x^2}=2x\times e^{x^2}=2x\,e^{x^2}$

**Example :** $q(t)=q_0-q_0e^{\frac{-t}{CR}}$ CR = time constant, $q_0$ = maximum value of charge

$$\frac{d}{dt}q(t)=\frac{d}{dt}\left[q_0-q_0e^{-t/CR}\right]=\frac{d}{dt}q_0-\frac{d}{dt}q_0e^{-t/CR}=0-q_0\times-\frac{1}{CR}\times e^{-t/CR}=\frac{q_0}{CR}e^{-t/CR}$$

**Example :** $f(x)=\frac{x}{(a^2+x^2)^{3/2}}$ where a=+ve constant.

$$\frac{d}{dx}f(x)=\frac{d}{dx}\frac{x}{(a^2+x^2)^{3/2}}=\frac{(a^2+x^2)^{3/2}\times\frac{d}{dx}x-x\times\frac{d}{dx}(a^2+x^2)^{3/2}}{\left[(a^2+x^2)^{3/2}\right]^2}$$

$$=\frac{(a^2+x^2)^{3/2}\times 1-x\times\frac{3}{2}(a^2+x^2)^{3/2-1}\times(0+2x)}{(a^2+x^2)^3}=\frac{(a^2+x^2)^{3/2}-3x^2(a^2+x^2)^{1/2}}{(a^2+x^2)^3}$$

**Example :** $f(x)=2x^2+x$ $\frac{d}{dt}f(x)=\frac{d}{dx}f(x)\times\frac{dx}{dt}=\frac{d}{dx}(2x^2+x)\times\frac{dx}{dt}=(4x+1)\times\frac{dx}{dt}$

**Example :** Find first derivative of y w.r.t. x.

(i) $y=e^{-x}$ (ii) $y=4\sin 3x$ (iii) $y=4e^{x^2-2x}$

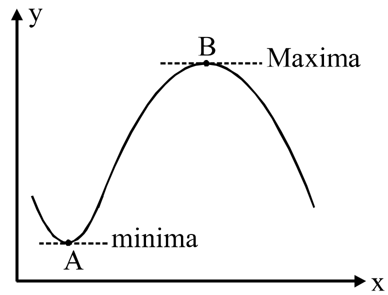
**Solution:** (i) $y=e^{-x}=e^z$ where $z=-x$ so $\frac{dy}{dx}=\frac{dy}{dz}\times\frac{dz}{dx}=\left(e^z\right)\left(-1\right)=-e^z=-e^{-x}$

(ii) $y=4\sin 3x=4\sin z$ where $z=3x$ so $\frac{dy}{dx}=\frac{dy}{dz}\times\frac{dz}{dx}=4\left(\cos z\right)\left(3\right)=12\cos 3x$

(iii) $y=4e^{x^2-2x}=4e^z$ where $z=x^2-2x$ so $\frac{dy}{dx}=\frac{dy}{dz}\times\frac{dz}{dx}=4\left(z\right)\left(2x-2\right)=\left(8x-8\right)e^{x^2-2x}$

## ● MAXIMUM AND MINIMUM OF A FUNCTION :

Higher order derivatives are used to find the maximum and minimum values of a function. At the points of maxima and minima, first derivative becomes zero.




**At point 'A' (minima) :** As we see in figure, in the neighbourhood of A, slope is increases so $\frac{d^2y}{dx^2} > 0$.

**Condition for minima :** $\boxed{\frac{dy}{dx} = 0 \text{ and } \frac{d^2y}{dx^2} > 0 \text{ or } \frac{d^2y}{dx^2} = +ve}$

**At point 'B' (maxima) :** As we see in figure, in the neighbourhood of B, slope is decreases so $\frac{d^2y}{dx^2} < 0$

**Condition for maxima :** $\boxed{\frac{dy}{dx} = 0 \text{ and } \frac{d^2y}{dx^2} < 0 \text{ or } \frac{d^2y}{dx^2} = -ve}$

**Example :** Find the minimum value of $y = 5x^2 - 2x + 1$ ?

**Solution :** For maximum/minimum value $\frac{dy}{dx} = 0 \Rightarrow 5(2x) - 2(1) + 0 = 0 \Rightarrow x = \frac{1}{5}$

Now at $x = \frac{1}{5}$, $\frac{d^2y}{dx^2} = 10$ which is positive so minima at $x = \frac{1}{5}$.

Therefore $y_{min} = 5\left(\frac{1}{5}\right)^2 - 2\left(\frac{1}{5}\right) + 1 = \frac{4}{5}$

## 2.3.2 INTEGRATION CALCULUS

- Integration means summation or addition.

$$\int f(x)dx = \text{integration of } f(x) \text{ with dx}$$

- $\int vdt =$ displacement
- $\int adt =$ velocity
- $\int Pdt =$ work
- $\int Fdx =$ work
- $\int Eds = \phi$ (Electric flux)
- Integration is the reverse process of differentiation.

If $\int f(x)dx = g(x)$ then $\frac{d}{dx}g(x) = f(x)$

$$A \underset{\text{Differentiation}}{\overset{\text{Intergration}}{\rightleftarrows}} B$$

## • RULES FOR SIMPLE INTEGRATION

- $f(x) = x^n \quad n \neq -1$ $\qquad \int f(x)dx = \int x^n dx = \frac{x^{n+1}}{n+1}$
- $f(x) = \frac{1}{x} = x^{-1}$ $\qquad \int f(x)dx = \int \frac{1}{x}dx = \log_e x = \ln x$
- $f(x) = \text{constt.}\, x^n$ $\qquad \int \text{constt.}x^n dx = \text{constt}\frac{x^{n+1}}{n+1}$
- $f(x) = \text{constant}$ $\qquad \int \text{constant} \cdot dx = \int \text{constant} \cdot x^0 dx = \text{constt} \times \frac{x^{0+1}}{0+1} = \text{constt.}x$
- $f(x) = \sin x$ $\qquad \int \sin x\, dx = -\cos x$
- $f(x) = \cos x$ $\qquad \int \cos x\, dx = \sin x$
- $f(x) = e^x$ $\qquad \int e^x dx = e^x$
- $f(x) = \sin ax$ $\qquad \int \sin ax\, dx = \frac{1}{a} \times -\cos ax$
- $f(x) = \cos ax$ $\qquad \int \cos ax\, dx = \frac{1}{a} \times \sin ax$
- $f(x) = e^{ax}$ $\qquad \int e^{ax} dx = \frac{1}{a}e^{ax}$
- $f(x) = \frac{1}{(a+bx)}$ $\qquad \int \frac{1}{(a+bx)}dx = \frac{1}{b}\log_e(a+bx)$
- $\int [f(x) \pm g(x)]dx = \int f(x)dx \pm \int g(x)dx$

Integration

- Definite Integration [When limit of Integration is taken]
- Indefinite Integration [When limit of Integration is not taken]

## • (i) DEFINITE INTEGRATION

- When a function is integrated between a lower limit and an upper limit, it is called a definite integration. Consider a function f(x) whose integration with dx is equal to F(x) , in an interval $a \leq x \leq b$ then

$$\int_a^b f(x)dx = F(b) - F(a)$$

**Example :** $\int_{x_1}^{x_2} x^n dx = \left[\frac{x^{n+1}}{n+1}\right]_{x_1}^{x_2} = \left[\frac{x_2^{\,n+1}}{n+1} - \frac{x_1^{\,n+1}}{n+1}\right]$

**Example :** $\int_0^{\pi/2} \sin\theta\, dx = [-\cos\theta]_0^{\pi/2} = \left[\left(-\cos\frac{\pi}{2}\right) - (-\cos 0)\right] = [0-(-1)] = 1$

**Example :** $\int_a^b \frac{1}{r} dr = [\log_e r]_a^b = [\log_e b - \log_e a] = \log_e \frac{b}{a}$

## • (ii) INDEFINITE INTEGRATION

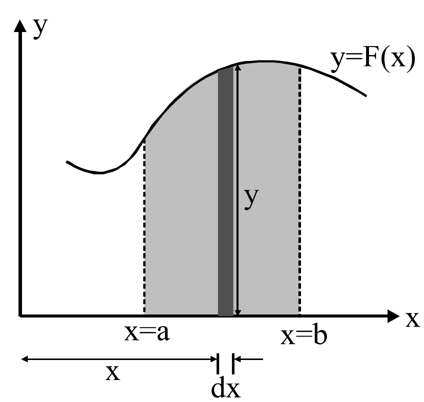
- When there is no upper and lower limit. By help of indefinite integration we can find a function whose derivative is known. Consider a function f(x) whose integration with dx is equal to F(x) then

$$\boxed{\int f(x)dx = F(x) + C}$$

**Example:** $\int x^n dx = \frac{x^{n+1}}{n+1} + C$ C = constant

**Example:** $\int \cos^2 \omega t \cdot dt = \int \frac{1+\cos 2\omega t}{2} dt = \frac{1}{2}\int (1+\cos 2\omega t)dt = \frac{1}{2}\left[t + \frac{1}{2\omega} \times -\sin 2\omega t + C\right] = \frac{1}{2}\left[t - \frac{\sin 2\omega t}{2\omega} + C\right]$

## • AREA UNDER A CURVE & DEFINITE INTEGRATION




Area of small shown element = ydx = f(x) dx

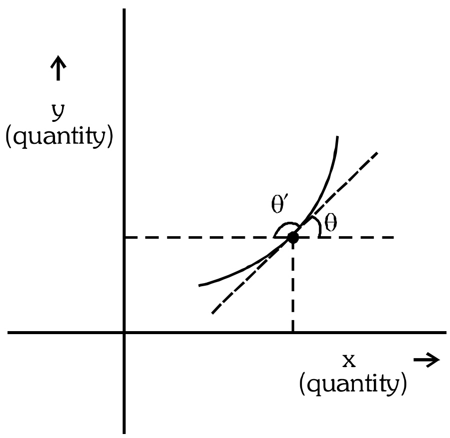
If we sum up all areas between x=a and x= b then $\int_a^b f(x)dx$ = shaded area between curve and x-axis.

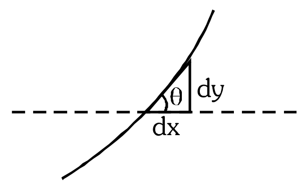
## 2.4 CO-ORDINATE GEOMETRY

- Graph is diagrammatic representation of a function and allows us to visualize it, actually it is the diagrammatic illustration of variation of a physical quantity with another physical quantity.

### • SLOPE OF THE GRAPH (m)

- Slope or gradient is the steepness of the line or curve at a particular point.



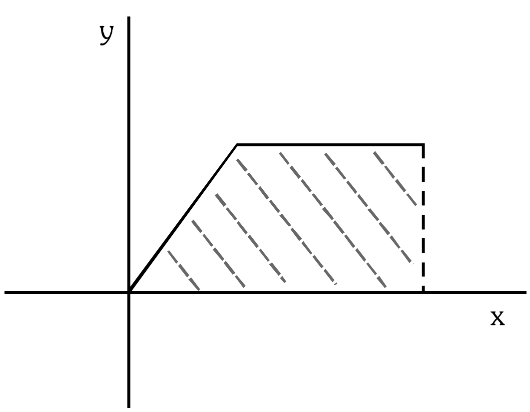

slope of the graph at any point is, $\boxed{m = \tan\theta = \frac{dy}{dx}}$

also, slope $m = \tan(180^0 - \theta') = -\tan\theta'$

- If $\theta < 90^0$ (acute angle), then $\tan\theta = +ve$ therefore, slope $m = +ve$ .
- If $\theta > 90^0$ (obtuse angle), then $\tan\theta = -ve$ therefore, slope $m = -ve$ .
- If $\theta$ increases from $0^0$ to $90^0$ then slope will be positive and value of slope increases.
- If $\theta$ decreases from $180^0$ to $90^0$ then slope will be negative and value of slope increases.
- If $\theta$ is constant then slope will also be constant.

### • AREA UNDER GRAPH

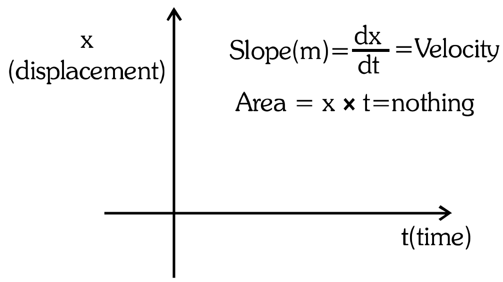
- The area under graph (+ve or –ve) gives the product of those two quantities between which the graph is drawn.

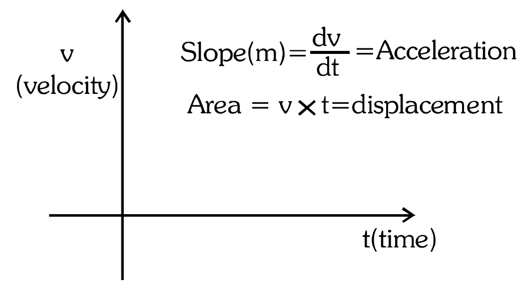


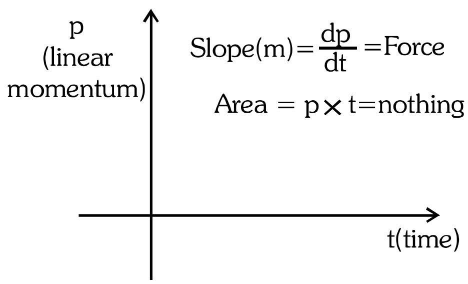

Area under graph $= x \times y$

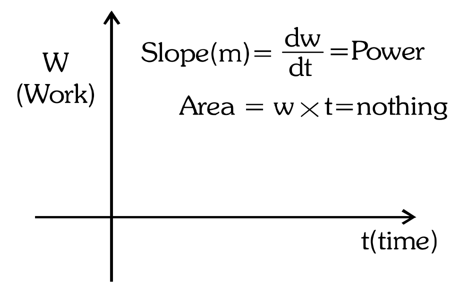
## • NOTE

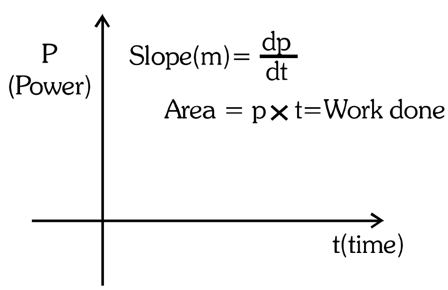
- किसी भी दिये हुऐ graph से 3 information हासिल की जा सकती है–
  (1) Relation between two Quantities.
  (2) Variation of a 3rd Quantity by its slope.
  (3) Value of another quantity by the area under graph.

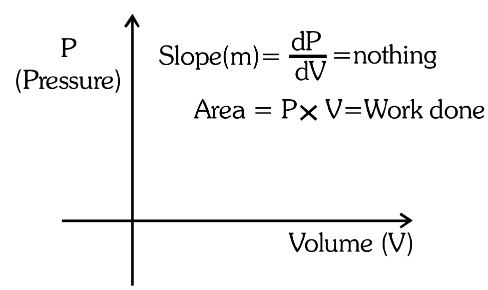


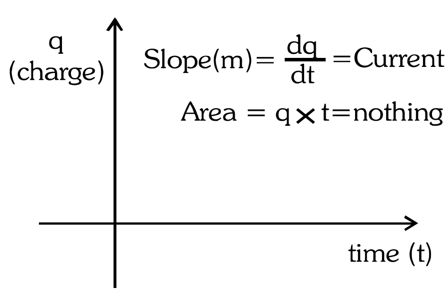



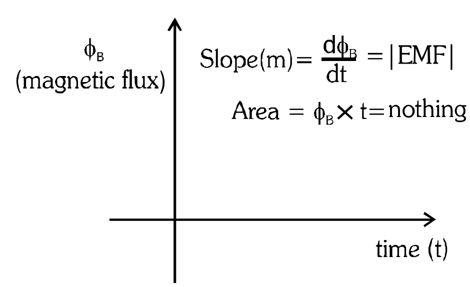




## • HOW TO DRAW A GRAPH BY GIVEN INFORMATION OR EQUATION

- To draw the graph find the relation between x and y without any other variable and then :-

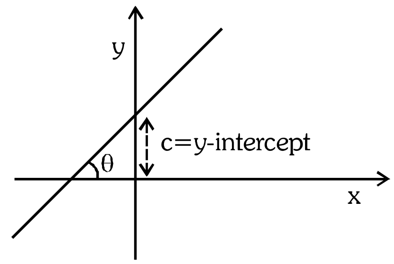
  $x^1$ $y^1$ = straight line $x^2$ $y^1$ or $x^1$ $y^2$ = Parabola

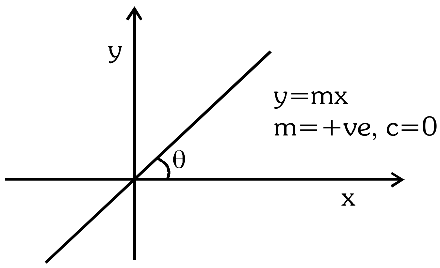
  $x^2$ $y^2$ = Circle or ellipse

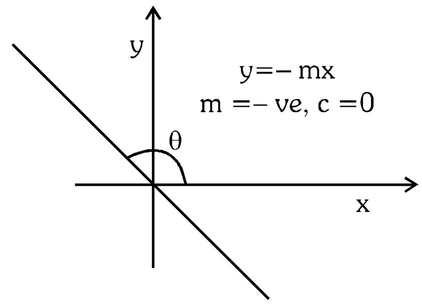
## ● STRAIGHT LINE

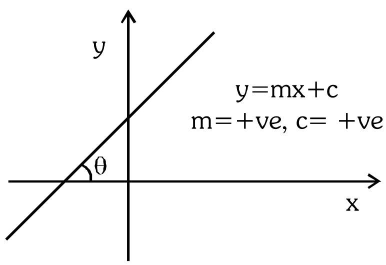
General equation : $y = mx + c$

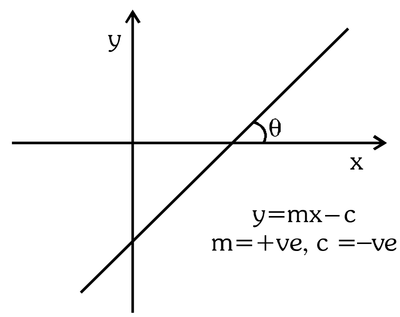
$y$ = Quantity on $y$ - axis
$x$ = Quantity on $x$ - axis
$m$ = slope
$c$ = interception of straight line on $y$ - axis

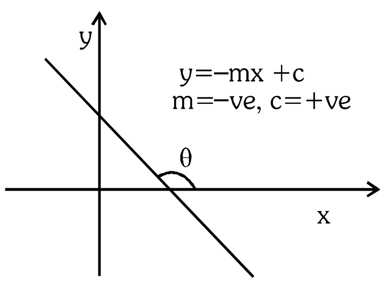


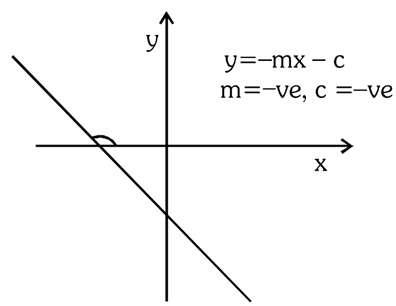



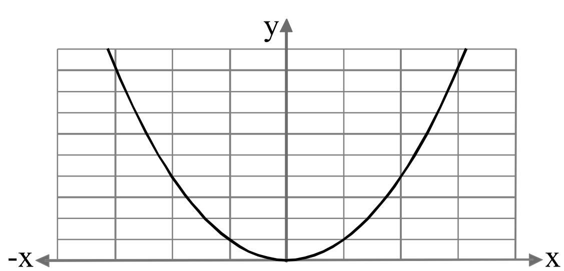



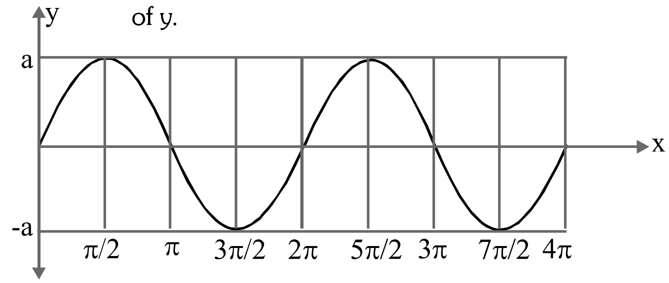

## ● PARABOLA

A function of the form $y = ax^2 + bx + c$ is known as parabola. The simplest parabola has the form $y = ax^2$



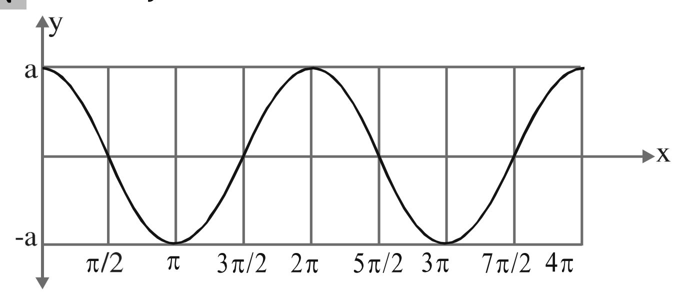
## Sine FUNCTION

$y = a \sin x$ Here, a is known as the amplitude and equals to the maximum magnitude of y.



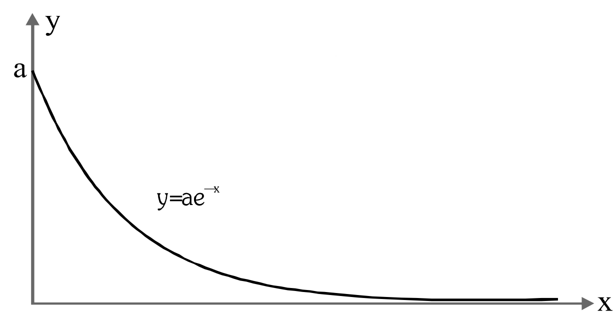

## Cosine FUNCTION

$y = a \cos x$




## EXPONENTIAL FUNCTION

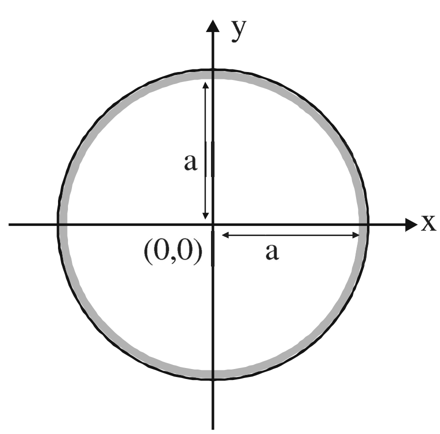
Behaviour of several physical phenomena is described by exponential function to the base e. Here e is known as Euler's Number. e=2.718218

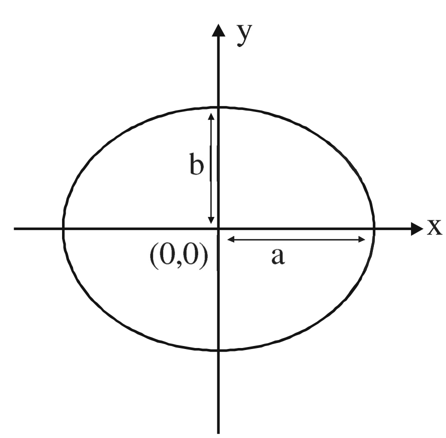



## CIRCLE & ELLIPSE

Circle : $x^2 + y^2 = a^2$

Ellipse : $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$



# CHAPTER 4 : MOTION IN STRAIGHT LINE AND MOTION IN A PLANE

- Kinematics is the branch of physics concerned with the motion of object without reference to the forces which causes the motion.
- Motion is a relative term which signifies change in position or orientation of body with time with respect to a reference point or axis.
- The motion may be translatory motion,or rotational motion combination of bothor .
- Translatory motion, is a type of motion in which all part of body, move same distance in same direction, in a given time.

**TRANSLATORY MOTION**

**MOTION IN ONE DIMENSION (1-D)**
(Only one coordinate change with time)
**E.g. Rectilinear motion or motion in straight line**

**MOTION IN TWO DIMENSION (2-D) OR MOTION IN A PLANE**
(Only two coordinates simultaneously change with time )
**E.g. Projectile motion along parabola, Circular motion, Curvilinear motion**

## 3.1 ELEMENTS OF TRANSLATORY MOTION

### • DISTANCE

- It is the total path length travelled by the body during a journey.
- It is a scalar quantity, i.e. having magnitude only.
- Distance can never decrease with time.
- Distance cannot be $-$ve in sign.
- Distance -time graph cannot lie in negative quadrant i.e. lies only in first quadrant.

### • DISPLACEMENT

- It is the vector representing the shortest length between initial and final point for a journey.
- It starts from initial point and ends to final point.
- It may be negative with reference to its direction therefore displacement -time graph may lie in first and fourth quadrant.
- It may decrease with time, i.e, body is returning towards initial point.
- Distance depends on path while displacement is independent of path but depends only on final and initial position.
- For an instant distance and displacement are having same magnitude but for a time interval distance may be equal to or greater than the magnitude of displacement.

$$\text{distance} \geq \text{displacement} \qquad \Rightarrow \qquad \frac{\text{distance}}{\text{displacement}} \geq 1$$

- If the displacement is zero for a journey the distance may or may not be zero but if the distance is zero, the displacement will surely be zero.
- In a journey, it is possible that displacement is zero but velocity is never zero for example, in one revolution of circular motion. But in straight line motion if displacement is zero for a journey then velocity at some instant must be zero.

## • SPEED (INSTANTANEOUS SPEED)

- The rate of change of distance at any instant during a journey is called as speed (v).

$$v = \frac{dx}{dt} \quad \text{OR} \quad v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

**Unit :** $ms^{-1}$ **Dimension :** $[LT^{-1}]$

- Speed is a scalar quantity i.e. having magnitude only.
- It cannot be $-$ve in sign at all therefore speed -time graph never lies in negative quadrant, only lies in first quadrant.
- The speed may increase or decrease with time as motion is accelerated or retarded.

## • AVERAGE SPEED

- It is the ratio of total distance to the total time for a certain journey.

$$\text{Average speed}(v_{av}) = \frac{\text{total distance}}{\text{total time}}$$

- If average speed is zero for a journey, i.e. body is at rest and instantaneous speed is also zero.
- For a journey instantaneous speed may be equal to or greater than or smaller than average speed.
- If the body is moving with constant speed between any two point then instantaneous speed and average speed will be same.

## • VELOCITY (INSTANTANEOUS VELOCITY)

- The rate of change of displacement at any instant during a journey, is called as instantaneous velocity $(\vec{v})$.

$$\vec{v} = \frac{d\vec{x}}{dt} \quad \text{OR} \quad \vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{x}}{\Delta t}$$

**Unit :** $ms^{-1}$ **Dimension :** $[LT^{-1}]$

- It is a vector quantity and directed along the direction of displacement at that instant.
- Velocity may be $+$ve or $-$ve in sign according to the reference of direction therefore velocity -time graph may lie in negative quadrant.
- The magnitude of velocity may increase or decrease with time as the motion is accelerated or retarded.
- The value (magnitude) of instantaneous velocity is equal to instantaneous speed as for an instant distance and displacement are same. In other words speed is the magnitude of velocity.
- A body may have varying velocity without having varying speed. (If only direction changes not the magnitude like Uniform Circular Motion)
- If the speed is varying then the velocity will surely vary.
- It is possible that a body is at rest but still its velocity is changing (This sentence is valid for an instant only not for a time interval) E.g. : When a body is thrown vertically upward then at highest point for an instant the body is at rest but the direction of velocity is changing.

## • AVERAGE VELOCITY

- It is the ratio of total displacement to the total time for a journey.

$$\text{Average velocity } (\vec{v}_{av}) = \frac{\text{total displacement}}{\text{total time}}$$

- It is a vector quantity and directed along net displacement i.e. from initial point to final point.
- The value of average speed may be greater or equal to average velocity.

$\because$ Total distance $\geq$ Net displacement

$\therefore$ Average speed $\geq$ | Average velocity |

$$\frac{\text{average speed}}{|\text{average velocity}|} \geq 1$$

- If the average velocity for a journey is zero, then average speed may or may not be zero. If the body does not move average velocity and average speed both will be zero and if the body reaches to its initial point after travelling some path length, average velocity is zero but average speed is not zero.
- If the average speed for a journey is zero, then average velocity is surely zero.
- If the average velocity in a journey is zero, then it may be possible that instantaneous velocity is never zero. For example during one revolution of circular motion.
- In straight line motion (1-D motion), if the average velocity in a journey is zero then, the instantaneous velocity is surely zero at some instant as the body has returned to its initial point.

## NCERT EXAMPLE

**Example 3.1** A car is moving along a straight line, say OP in Fig. 3.1. It moves from O to P in 18 s and returns from P to Q in 6.0 s. What are the average velocity and average speed of the car in going (a) from O to P ? and (b) from O to P and back to Q ?

**Answer :** $\text{Average velocity} = \dfrac{\text{displacement}}{\text{time interval}}$

$$\bar{v} = \frac{+360\text{m}}{18\text{s}} = +20\text{m/s}$$

$$\text{Average speed} = \frac{\text{Path length}}{\text{time interval}} = \frac{360\text{m}}{18\text{s}} = 20\text{m/s}$$

Thus, in this case the average speed is equal to the magnitude of the average velocity.

(b) In this case,

$$\text{Average velocity} = \frac{\text{displacement}}{\text{time interval}} = \frac{+240\text{m}}{(18+6.0)\text{s}} = 10\text{m/s}$$

$$\text{Average speed} = \frac{\text{Path length}}{\text{time interval}} = \frac{OP+PQ}{\Delta t} = \frac{(360+120)\text{m}}{24\text{s}} = 20\text{m/s}$$

## • ACCELERATION

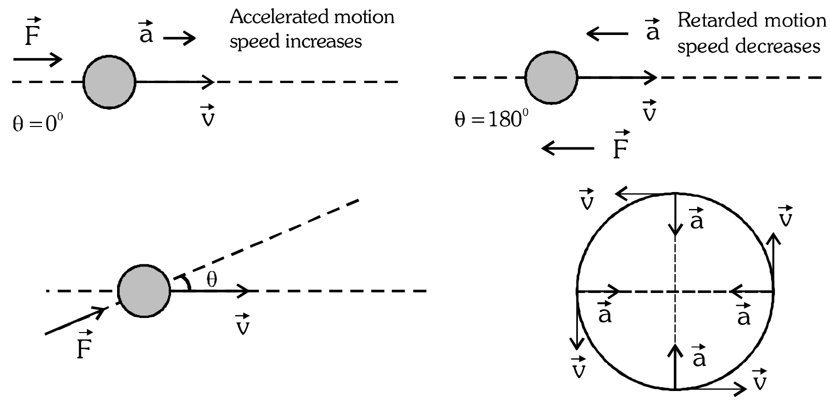
- Acceleration $(\vec{a})$ is the rate of change of velocity at any instant and average acceleration $(\vec{a}_{av})$ is the ratio of change in velocity over time interval during a journey.

$$\vec{a} = \frac{d\vec{v}}{dt} \qquad \vec{a}_{av} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_{final} - \vec{v}_{initial}}{\Delta t}$$

**Unit :** $m/s^2$ **Dimension :** $[LT^{-2}]$

- The acceleration is produced in a body only when net force is applied on it.
- The direction of acceleration is same as direction of force and it does not depend on the direction of velocity or motion (the direction of acceleration can be found by finding the direction of change in velocity).
- The body will have non-zero acceleration only when there is change in velocity (its magnitude or direction or both changes) and if the body is moving with constant velocity acceleration will be zero.
- If the acceleration is constant, it means its magnitude as well as direction both are constant. If the acceleration is variable it means either magnitude or direction or both are variable.
- The motion is said to be retarded only when the magnitude of velocity or speed decreases.

- If the force is applied at angle $0^0$ or $180^0$ with velocity (motion), the body will move on straight line performs 1-D motion and the motion will be accelerated or retarded respectively.
- If the force is applied perpendicular ($90^0$) to the motion, speed or magnitude of velocity does not change but velocity will change as the direction changes. The acceleration produced, is non-uniform (as the direction of acceleration changes continuously). E.g. Centripetal acceleration during Uniform Circular Motion.
- If the force is applied at angle other than $0^0$, $180^0$ or $90^0$ then path of body will be PARABOLA.




- If acceleration is constant then it is possible that the direction of velocity gets reversed.
  For example: During Straight line motion under gravity the acceleration is always directed towards downward but the direction of motion gets reversed from upper to downward.
- If the acceleration is decreasing then it does not mean that motion is retarded, actually speed will be increasing but at a lower rate.
- It is possible for a body to have non-zero acceleration without having varying speed, and this acceleration will be variable like in Uniform Circular Motion.
- If body is at rest at some instant then it may have non zero acceleration but if the body is at rest for time interval then its acceleration will surely be zero.
- Acceleration may be positive or negative according to its direction.
- If the sign of acceleration and velocity are same then motion will be accelerated and if they are having opposite sign then motion will be retarded.

| | | |
|---|---|---|
| If $\vec{v} = +ve$ | $\vec{a} = +ve$ | Motion is accelerated |
| If $\vec{v} = -ve$ | $\vec{a} = -ve$ | Motion is accelerated |
| If $\vec{v} = +ve$ | $\vec{a} = -ve$ | Motion is retarded |
| If $\vec{v} = -ve$ | $\vec{a} = +ve$ | Motion is retarded |

## NCERT EXAMPLE

**Example 3.2** The position of an object moving along x-axis is given by $x = a + bt^2$ where a = 8.5 m, b = 2.5 m s–2 and t is measured in seconds. What is its velocity at t = 0 s and t = 2.0 s. What is the average velocity between t = 2.0 s and t = 4.0 s ?

**Answer** In notation of differential calculus, the velocity is $v = \frac{dx}{dt} = \frac{d}{dt}\left(a + bt^2\right) = 2bt = 5.0t\,m/s$

At t=0 s, v=0 m/s and at t=2.0 s,

$$\text{Average velocity} = \frac{x(4.0) - x(2.0)}{4.0 - 2.0} = \frac{a + 16b - a - 4b}{2.0} = 6.0 \times b = 6.0 \times 2.5 = 15 m/s$$

## 3.2 MOTION IN ONE DIMENSION OR MOTION IN STRAIGHT LINE

**Motion in straight line (Rectilinear Motion)**

- **Uniform Velocity Motion**
- **Accelerated Motion**
  - **Uniform Acceleration Motion**
  - **Variable Acceleration Motion**
    - **I. Acceleration as function of time.**
    - **II. Acceleration as function of position.**
    - **III. Acceleration as function of velocity.**

## 3.2.1 MOTION WITH UNIFORM VELOCITY (UNIFORM MOTION)

- In uniform velocity motion, a body moves with constant speed on a straight–line path without change in direction.
- When a particle moves with constant velocity, then the magnitude of average velocity, magnitude of instantaneous velocity, average speed and its speed all are equal.

### • FORMULAE FOR THIS MOTION

$$\text{velocity} = \frac{\text{displacement}}{\text{time}} \qquad \boxed{\vec{v} = \frac{\vec{s}}{t}} \qquad \text{OR} \qquad \text{speed} = \frac{\text{distance}}{\text{time}} \qquad \boxed{v = \frac{s}{t}}$$

### • NUMERICAL TIPS

- In a journey, if the body travels with different constant speeds in different parts of journey then we have to study those parts separately.

## 3.2.2 MOTION WITH UNIFORM OR CONSTANT ACCELERATION

- Motion in which acceleration remains constant in magnitude as well as direction is called uniform accelerated motion.
- If the force acting on body is constant then acceleration will be constant, the body will be accelerated or retarded uniformly in one dimension motion.

### • FORMULAE FOR THIS MOTION

- If the body is moving with uniform acceleration following equations of motion are used-

$$\boxed{\vec{v} = \vec{u} + \vec{a}\Delta t} \qquad \boxed{\vec{s} = \vec{u}\Delta t + \frac{1}{2}a(\Delta t)^2} \qquad \boxed{v^2 = u^2 + 2\vec{a}.\vec{s}} \qquad \boxed{\vec{S}_n = \vec{u} + \frac{1}{2}\vec{a}(2n-1)}$$

$\vec{u}$ = Initial velocity i.e. velocity at an instant from where equation is applied (say t = 0).

$\vec{v}$ = Final velocity at an instant upto which equation is applied (say t = t).

$\Delta t$ = Time interval for which equation is applied.

$\vec{s}$ = Displacement during the interval t = 0 to t = t (it is equal to distance if the body does not alter its direction)

$\vec{S}_n$ = Displacement in $n^{th}$ second (a particular second).

$\vec{a}$ = Acceleration produced in body.

- **Sign convention for equations of motion:** You can take any direction as positive and just opposite direction as negative and then assign the sign to different quantities. For example we can take the sign of $(\vec{v}, \vec{a}, \vec{S})$ with reference to initial velocity $\vec{u}$ i.e. if these vectors are in the direction of $\vec{u}$ then their sign will be positive and if just opposite to $\vec{u}$ then taken as negative. If initial velocity is zero all the vectors have positive sign.
- **MOTION IN STRAIGHT LINE UNDER GRAVITY**
  Suppose upward direction is +ve and downward direction is −ve .

**Example I :** Journey from A to B (upward journey)

u = +ve  v = + ve  h = +ve  a = g = −ve

$v = u - gt$  $h = ut - \frac{1}{2}gt^2$  $v^2 = u^2 - 2gh$

**Example II :** Journey from B to A (downward journey)

u = −ve  v = −ve  h = −ve  a = g = −ve

$-v = -u - gt$  $-h = -ut - \frac{1}{2}gt^2$  $(-v)^2 = (-u)^2 + 2 \times (-g) \times (-h)$

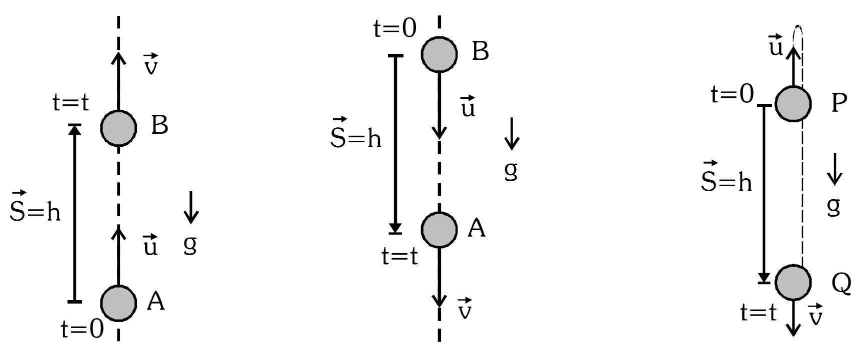
$v = u + gt$  $h = ut + \frac{1}{2}gt^2$  $v^2 = u^2 + 2gh$

**Example III :** Journey from P to Q (upward motion at initial point and downward motion at final point)

u = +ve  v = −ve  h = −ve  a = g = −ve

$-v = u - gt$  $-h = ut - \frac{1}{2}gt^2$  $(-v)^2 = (u)^2 + 2 \times (-g) \times (-h)$

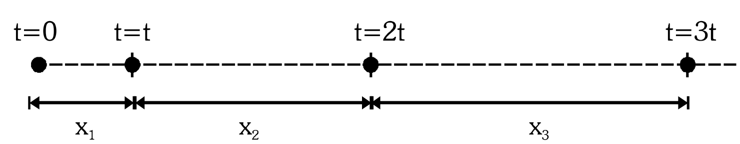
$v = -u + gt$  $h = -ut + \frac{1}{2}gt^2$  $v^2 = u^2 + 2gh$




**(I)** **(II)** **(III)**

## • NUMERICAL TIPS

- **OBSERVATION I :** If a body starts from rest i.e. the initial velocity is zero and motion is uniformly accelerated then distance travelled in subsequent equal time interval are in the ratio 1 : 3 : 5 : 7 ........
  (Glileo's law of odd number)



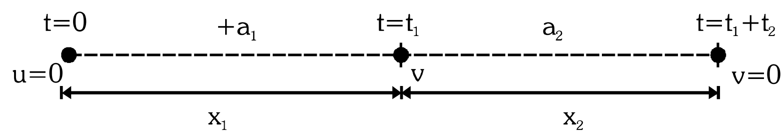

$x_1 : x_2 : x_3 = 1 : 3 : 5$

t=0 to t =t $\quad x_1 = 0 + \frac{1}{2}at^2 \quad x_1 = \frac{1}{2}at^2$

t =0 to t = 2t $\quad u = 0 \quad x_1 + x_2 = 0 \times 2t + \frac{1}{2}a(2t)^2$

$x_1 + x_2 = 4 \times \frac{1}{2}at^2 \quad x_2 = 4x_1 - x_1 \quad x_2 = 3x_1$

- **OBSERVATION II :** If a body starts from rest and again comes to rest i.e. first accelerated and then retarded, the distance travelled and time taken during accelerated and retarded motion has inverse relation with the magnitude of acceleration and retardation. And if the acceleration and retardation has same magnitude then distance travelled and time taken will be same.




$t = 0 \rightarrow t_1 \quad v = 0 + a_1t_1 \quad t_1 = \frac{v}{a_1} \quad v_2 = 0 + 2a_1x_1 \quad x_1 = \frac{v^2}{2a_1}$

$t = t_1 \rightarrow t = t_1 + t_2 \quad 0 = v - a_2t_2 \quad t_2 = \frac{v}{a_2} \quad 0 = v^2 - 2a_2x_2 \quad x_2 = \frac{v^2}{2a_2}$

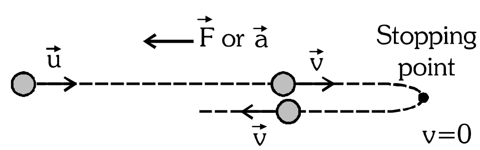
$\boxed{\frac{t_1}{t_2} = \frac{a_2}{a_1}} \quad \boxed{\frac{x_1}{x_2} = \frac{a_2}{a_1}} \quad \frac{x_1}{x_2} = \frac{t_1}{t_2} = \frac{a_2}{a_1} \quad x \propto \frac{1}{a} \quad t \propto \frac{1}{a}$

- **OBSERVATION III :** If a body starts with some initial velocity and a constant force in opposite direction is applied then the body first get retarded, comes to rest and then gets accelerated in just opposite direction with same acceleration. In this situation -

**(a)** The speed of body at a particular point is same while going towards stopping point and returning from stopping point.

**(b)** The distance travelled in a particular time interval before stopping and after stopping will be same.

**(c)** The time interval from a particular point to stopping point and from stopping point to the same point, are equal.



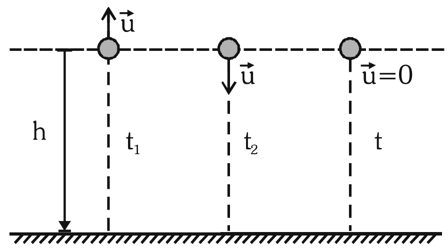

- **OBSERVATION IV :** If a body has initial velocity u and final velocity v then -

$v = u + at \quad at = v - u \quad t = \frac{v-u}{a}$

$v^2 = u^2 + 2aS \quad 2aS = v^2 - u^2 \quad S = \frac{v^2 - u^2}{2a}$

Average velocity $= \frac{S}{t} = \frac{(v^2 - u^2) \times a}{2a \times (v-u)} = \frac{(v+u)(v-u)}{2(v-u)} \quad \boxed{v_{avg} = \frac{v+u}{2}}$

- **OBSERVATION V :** If the different bodies have same speed at the same level from the ground then the final speed on reaching the ground will also be same whatever is the mass of body or path, time taken to reach the ground may be different.
- Let a three bodies are at same level,one is thrown downward with some speed and another is thrown upward with same speed and the third is dropped then time taken by them to reach the ground will be different.The time taken are related as:




$h = -ut_1 + \frac{1}{2}gt_1^2$ ..........(i), $h = ut_2 + \frac{1}{2}gt_2^2$ ..........(ii), $h = \frac{1}{2}gt^2$ ..........(iii)

Equation (i) multiplies by $t_2$ and equation (ii) multiplied by $t_1$ and then added

$ht_2 = -ut_1t_2 + \frac{1}{2}gt_1^2t_2$ $ht_1 = ut_1t_2 + \frac{1}{2}gt_2^2t_1$

$h(t_1 + t_2) = \frac{1}{2}gt_1t_2(t_1 + t_2)$ $\Rightarrow$ $h = \frac{1}{2}gt_1t_2$ $\Rightarrow$ $h = \frac{1}{2}gt^2$

$h = \frac{1}{2}gt^2 = \frac{1}{2}gt_1t_2$ $\Rightarrow$ $t = \sqrt{t_1t_2}$

## NCERT EXAMPLE

**Example 3.4** A ball is thrown vertically upwards with a velocity of 20 m $s^{-1}$ from the top of a multistorey building. The height of the point from where the ball is thrown is 25.0 m from the ground. (a) How high will the ball rise? and (b) how long will it be before the ball hits the ground? Take g = 10 m $s^{-2}$.

***Answer*** (a) Let us take the *y*-axis in the vertically upward direction with zero at the ground. If the ball rises to height *y* from the point of launch, then using the equation

$v_0 = +20m/s$, $a = -g = -10ms^{-2}$, $v = 0ms^{-1}$

If the ball rises to height *y* from the point of launch, then using the equation

$v^2 = v_0^2 + 2a(y - y_0)$

we get $0 = (20)^2 + 2(-10)(y - y_0)$

Solving, we get, $(y - y_0) = 20m$

(b) We can solve this part of the problem in two ways. **Note carefully the methods used**.

## 3.2.3 MOTION WITH VARIABLE ACCELERATION

- In straight line motion acceleration may vary as a function of time, displacement or velocity.
- **Acceleration given as function of time :** If acceleration is a given function of time say a = f(t), from equation a = dv/dt, we have

  $dv = f(t)dt \Rightarrow \int dv = \int f(t)dt$ This equation expresses v as function of time, say v = g(t).

  Also, v = dx/dt, we have

  $dx = g(t)dt \Rightarrow \int dx = \int g(t)dt$ This equation expresses position or displacement as function of time.

- **Acceleration as function of position :** If acceleration is a given function of position say a = f(x), we have to use equation a = vdv/dx $\Rightarrow$ vdv = adx therefore we have,

  $vdv = f(x)dx \Rightarrow \int vdv = \int f(x)dx$ This equation expresses velocity as function of position v = g(x)

Also v = dx/dt, we have

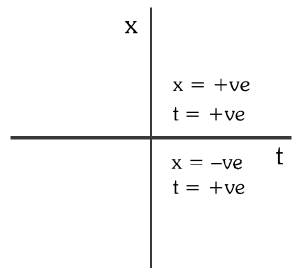
$$dt = \frac{dx}{g(x)} \Rightarrow \int dt = \int \frac{dx}{g(x)} = \int \frac{dx}{v(x)}$$ This equation expresses relation between x and t

- **Acceleration as function of velocity :** If acceleration is given as function of velocity say a=f(v), by using equation a = dv/dt and a = vdv/dx, we can obtain velocity as function of time.

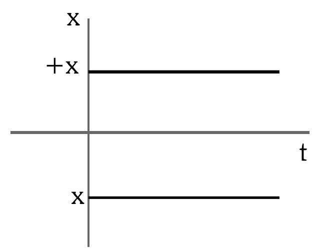
$$dt = \frac{dv}{f(v)} \Rightarrow \int dt = \int \frac{dv}{f(v)} \quad \text{and} \quad dx = \frac{vdv}{f(v)} \Rightarrow \int dx = \int \frac{vdv}{f(v)} = \int \frac{vdv}{a(v)}$$

## 3.2.4 GRAPHICAL APPROACH

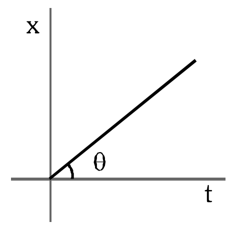
### • POSITION TIME/ DISPLACEMENT TIME GRAPH




Slope of x-t graph $(m) = \tan\theta = \frac{dx}{dt} = \text{velocity}$




(I) Position is not changing body is at rest



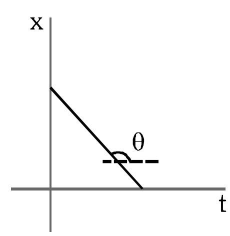

(II) x = +ve

$\because \theta < 90^0$ and $\theta$ is constant

$\Rightarrow$ slope = +ve and constant

$\Rightarrow$ velocity = +ve and constant

$\Rightarrow$ motion is uniform and unaccelerated. $(\vec{a} = 0)$



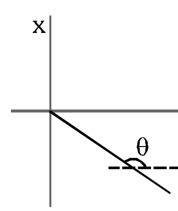

(III) x = +ve

$\because \theta > 90^0$ and $\theta$ is constant

$\Rightarrow$ slope = –ve and constant

$\Rightarrow$ velocity = –ve and constant

$\Rightarrow$ motion is unaccelerated




(IV) x = –ve

$\because \theta > 90^0$ and $\theta$ is constant

$\Rightarrow$ slope = –ve and constant

$\Rightarrow$ velocity = –ve and constant

$\Rightarrow$ Motion is unaccelerated

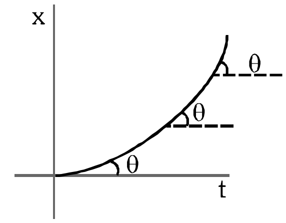
(V) x = +ve

∵ $\theta < 90^0$ and $\theta$ is increasing

⇒ slope = +ve and increasing

⇒ velocity = +ve and increasing

⇒ Motion is accelerated



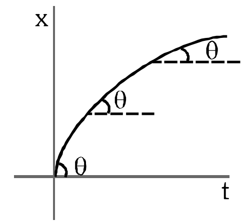

(VI) x = +ve

∵ $\theta < 90^0$ and $\theta$ is decreasing

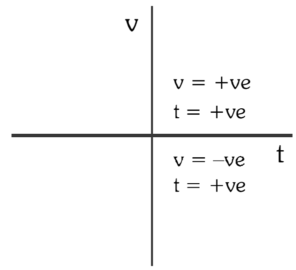
⇒ slope = + ve and decreasing

⇒ velocity = +ve and decreasing

⇒ Motion is retarded




## ● VELOCITY - TIME GRAPH

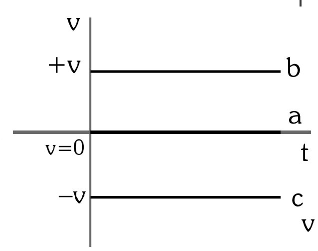



Slope of the graph $(m) = \tan\theta = \frac{dv}{dt}$ = acceleration

Area under graph $= v \times t$ = displacement

**If areas are added without sign then it gives distance also.**

(I) **a** - body is at rest
**b** - body is moving with constant (+ve) velocity
**c**- body is moving with constant (–ve) velocity



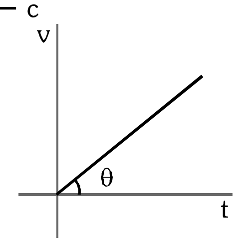

(II) v = +ve

∵ $\theta < 90^0$ and $\theta$ is constant

⇒ slope = +ve and constant

⇒ acceleration = +ve and constant

⇒ Motion is uniformly accelerated.



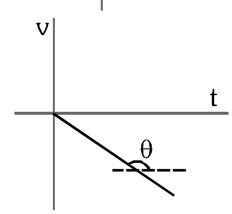

(III) v = –ve

∵ $\theta > 90^0$ and $\theta$ is constant

⇒ slope = –ve and constant

⇒ acceleration = –ve and constant

⇒ Motion is uniformly accelerated



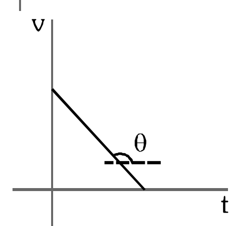

(IV) v = +ve

∵ $\theta > 90^0$ and $\theta$ is constant

⇒ slope = –ve and constant

⇒ acceleration = –ve and constant

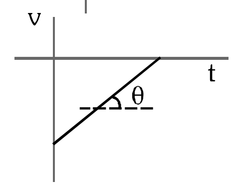
⇒ Motion is uniformly retarded




(V) v = –ve

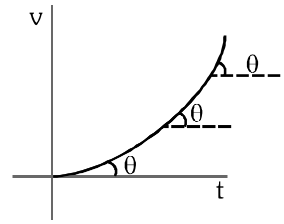
∵ $\theta < 90^0$ and $\theta$ is constant

⇒ slope = +ve and constant



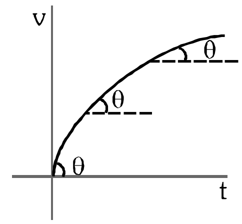
$\Rightarrow$ acceleration = +ve and constant
$\Rightarrow$ Motion is uniformly retarded.

(VI) v = +ve



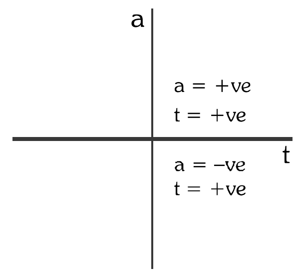

$\because$ $(\theta < 90^0)$ and increasing
$\Rightarrow$ slope = +ve and increasing
$\Rightarrow$ a = +ve and increasing
$\Rightarrow$ motion is non uniformly accelerated

(VII) v = +ve



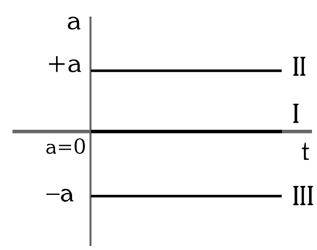

$\because$ $\theta < 90^0$ and $\theta$ is decreasing
$\Rightarrow$ slope = + ve and decreasing
$\Rightarrow$ a=+ve and decreasing
$\Rightarrow$ Motion is nonuniformly accelerated

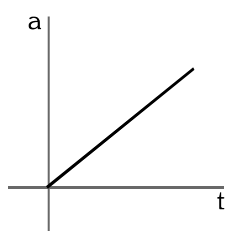
## • ACCELERATION - TIME GRAPH



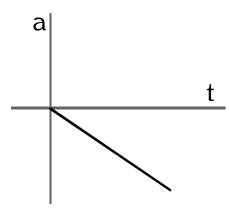

area under graph $= \vec{a} \times t = \vec{v} - \vec{u} =$ change in velocity

(I) a = 0 $\Rightarrow$ the motion is unaccelerated
(II) a = +ve and constant $\Rightarrow$ the motion is uniformly accelerated
(III) $a = -ve$ and constant $\Rightarrow$ the motion is uniformly retarded
(IV) a=+ve and increasing $\Rightarrow$ the motion is non-uniformly accelerated
(V) $a = -ve$ and increasing $\Rightarrow$ the motion in non-uniformly retarded



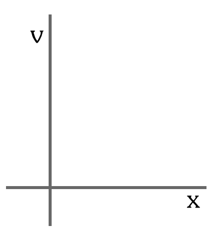




(IV)




(V)

## • NOTE

❍ 


$$\text{Slope (m)} = \tan\theta = \frac{dv}{dx} = \frac{\text{acceleration}}{\text{velocity}}$$