CHAPTER 1 : UNITS AND MEASUREMENT

1.1 PHYSICAL QUANTITIES

- Physics is the natural science that studies matter, its motion and behavior through space and time, and the related entities of energy and force.Physics is one of the most fundamental scientific disciplines, and its main goal is to understand how the universe behaves.
- A physical quantity is a property of a material or system that can be quantified by measurement. All the quantities which are used to describe the laws of physics are known as physical quantities.
- The magnitude of a physical quantity is the product of its numerical value and unit in which quantity is measured.
- Magnitude of a physical quantity = numerical value (n) $\times$ unit (u)

$$\boxed{nu = \text{constant} \quad \Rightarrow \quad n \propto \frac{1}{u}}$$

- On the basis of dependency for their definition physical quantities or units are of following 2 types :-

(1) FUNDAMENTAL OR BASE QUANTITIES OR UNITS :

- The quantities which do not depend upon other quantities for their complete definition are known as fundamental or base quantities. e.g. **Length, Mass, Time, etc.**
- The units of fundamental quantities are called as fundamental units.
- According to **FPS(Foot,Pound,Second) System, CGS(Centimeter,Gram,Second) System and MKS(Meter,Kilogram,Second) System** length, mass and time are taken as fundamental quantities.
- According to **SI system(International System)** there are 7 fundamental quantities and their units, which are as follows

Physical quantity	**Unit**	**Symbol**
Length	Metre	m
Mass	Kilogram	Kg
Time	Second	s
ThermodynamicTemperature	Kelvin	K
Electric current	Ampere	A
Luminous intensity	Candela	Cd
Amount of substance	Mole	mol

(2) DERIVED QUANTITIES OR UNITS :

- The quantities which depend upon other quantities for their complete definition and can be expressed in terms of the fundamental quantities, are known as derived quantities . e.g. : **Force, Work, Charge, etc.**
- The units of derived quantities are called as derived units.

POINTS TO REMEMBER (PTR)

- All derived quantities or units can be expressed in terms of fundamental quantities or units and also in terms of other derived quantities or units.

$$\underset{\text{work}}{\text{Joule}} = \underset{\text{mass}}{\text{Kg}}\ \underset{\text{length}}{\text{m}^2}\ \underset{\text{time}}{\text{sec}^{-2}} \qquad \underset{\text{work}}{\text{Joule}} = \underset{\text{force}}{\text{Newton}}\text{-}\underset{\text{length}}{\text{meter}}$$

- A fundamental quantity can be written in terms of derived quantities but a fundamental quantity cannot be written in terms of other fundamental quantities.

$$\text{Length} \leftarrow \text{Meter} = \frac{\text{Joule} \rightarrow \text{Work}}{\text{Newton} \rightarrow \text{Force}}$$

- **Supplementary Quantities :** There are following two supplementary quantities

Physical quantity	Unit	Symbol
Plane angle (θ)	Radian	Rad
Solid angle (ω)	Steradian	Sr

1.2 SOME UNITS OF MASS

(i) **1 Kilogram;** It is the mass of 1 litre water at 4^0C.

(ii) **1 Chandrashekhar limit (CL)** $= 1.4 \times$ mass of **Sun (Solar mass)**

(iii) **1 Atomic mass unit (amu)** $= 1.67 \times 10^{-27}$ kg

- The mass of Neutron and proton is almost equal to 1 amu.

1.3 SOME UNITS OF LENGTH

(i) **1 metre** : It is the length of 1650763.73 wavelengths of orange – red colour light emitted by Krypton (Kr)-86

OR It is the distance covered by light in air or vacuum during $\frac{1}{299,792,458}$ seconds

(ii) **1 Astronomical Unit (AU):** It is the average distance between Sun and Earth.

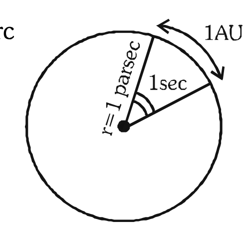
$$\boxed{1\,\text{AU} = 1.5 \times 10^{11}\ \text{m}}$$

- The distance between the Sun and the Earth is measured by parallax method.

(iii) **Light Year (LY):** It is the distance travelled by light in air or vacuum during 1 year (365 days).

$$\boxed{1\ \text{light year} = 3 \times 10^8 \times 365 \times 24 \times 60 \times 60 = 9.46 \times 10^{15}\ \text{m}}$$

(iv) **Parallactic Second (Par-Sec):** It is the radius of circle on which 1 AU arc subtends an angle of 1 second at centre.



$$\boxed{1\,\text{Par-sec} = 3.26\,\text{Light year} = 3.09 \times 10^{16}\ \text{m}}$$

(v) Some small units of length :

1 micron (μ) $= 10^{-6}$ m   1 nanometer (nm) $= 10^{-9}$ m   1 Angstrom ($\mathring{A}$) $= 10^{-10}$ m

1 picometer $= 10^{-12}$ m   1 fermi (fm) $= 10^{-15}$ m

- Par-sec is the largest unit of length and Fermi is the smallest unit of length.
- The order of size of atom is in angstrom (10^{-10} m) and order of size of nucleus is in Fermi (10^{-15} m).

1.4 SOME UNITS OF TIME

(i) 1 Second : It is the time taken by Cs – 133 atom to complete 9192631770 oscillations.

(ii) Solar Day : It is the time taken by the Earth to rotate about its axis so that the Sun appears at the same position in the sky.

(iii) Sidereal Day : It is the time taken by Earth to rotate about its axis so that a distant star other than Sun appears at the same position in the sky.

(iv) Lunar month : It is the time taken by Moon to complete one revolution around the Earth.
1 lunar month = 27.3 solar days

(v) Other unit : $1\,\text{shake} = 10^{-8}$ sec

- There is a difference of 3 minutes 56 seconds between solar day and sidereal day.

• PLANE ANGLE (θ) : It is the two dimensional angle drawn by bending two straight lines.

$$\text{Plane Angle} = \frac{\text{Length of arc (in meter)}}{\text{Radius(in meter)}} \text{ rad}$$

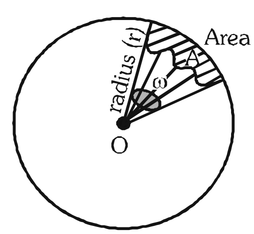
- Total Plane angle subtended by circle $= \dfrac{\text{Total length of arc}}{\text{Radius}} = \dfrac{2\pi \cancel{r}}{\cancel{r}} = 2\pi(\text{rad})$

$2\pi \text{ radian} = 360^0$ $\pi \text{ rad} = 180^0$ $\boxed{1\,\text{Radian} = \dfrac{180^0}{\pi}}$ $\boxed{1^0 = \dfrac{\pi}{180^0}\text{radian}}$

Example : $45^0 = \dfrac{\cancel{45}^0}{\cancel{180}^0}\dfrac{1\times\pi}{4} = \dfrac{\pi}{4}\text{rad}$ $120^0 = \dfrac{\cancel{120}^0}{\cancel{180}^0}\dfrac{2\times\pi}{3} = \dfrac{2\pi}{3}\text{rad}$

• SOLID ANGLE (ω) : It is the three dimensional angle subtended by certain area at the centre.

$$\omega = \frac{\text{Area (in SI unit)}}{\text{radius}^2\text{(in SI unit)}} \text{ steradian}$$



Shell

- Total solid angle subtended by shell $= \dfrac{\text{Total surface area}}{\text{radius}^2} = \dfrac{4\pi r^2}{r^2} = 4\pi\,\text{Sr.}$

NCERT EXAMPLE

Example 2.5 : If the size of a nucleus (in the range of 10^{-15} to 10^{-14} m) is scaled up to the tip of a sharp pin, what roughly is the size of an atom ? Assume tip of the pin to be in the range 10^{-5}m to 10^{-4}m.

Answer : The size of a nucleus is in the range of 10^{-15} m and 10^{-14} m. The tip of a sharp pin is taken to be in the range of 10^{-5} m and 10^{-4} m.Thus we are scaling up by a factor of 10^{10}. An atom roughly of size 10^{-10} m will be scaled up to size of 1 m. Thus a nucleus in an atom is as small in size as the tip of a sharp pin placed at the centre of a sphere of radius about a metre long.

1.5 DIMENSIONS

- The dimensions of a physical quantity are the powers (or exponents) to which the base quantities or units are raised to represent that quantity.

Example : In work or Joule the dimensions of mass, length and time are 1, 2, –2 respectively

Joule = kg m^2 sec^{-2}

Joule → work; kg → mass^1; m^2 → length^2; sec^{-2} → time^{-2}

- The dimensional formula or dimensional equation of any physical quantity is that expression which represents how and which of the base or fundamental quantities are included in that quantity.
- It is written by enclosing the following symbols for base quantities with appropriate powers in square brackets i.e.
  Mass [M] Length [L] Time [T] Current [A] Temperature [θ or K]

• DIMENSIONAL FORMULA OF SOME GENERAL PHYSICAL QUANTITIES

- Velocity $= \dfrac{\text{displacement}}{\text{time}} = \dfrac{[L]}{[T]} = [LT^{-1}]$
- Acceleration $= \dfrac{\Delta v}{\Delta t} = \dfrac{[LT^{-1}]}{[T]} = [LT^{-2}]$
- Force = ma $= [MLT^{-2}]$
- Linear momentum (p) $= mv = [MLT^{-1}]$
- Impulse $= F \times \Delta t = [MLT^{-2}] \times [T] = [MLT^{-1}]$
- Work or Energy or Heat $= F \times S = [MLT^{-2} \times L] = [ML^2T^{-2}]$
- Torque $(\tau) = F \times d = [MLT^{-2} \times L] = [ML^2T^{-2}]$
- Power (P) $= \dfrac{W}{t} = \dfrac{[ML^2T^{-2}]}{[T]} = [ML^2T^{-3}]$
- Frequency or angular velocity $(\omega) = \dfrac{1}{T}$ or $\dfrac{2\pi}{T} = [T^{-1}]$
- Moment of Inertia (I) $= mr^2 = [ML^2]$
- Angular momentum (J) $= I\omega = [ML^2T^{-1}]$
- Pressure or stress $= \dfrac{F}{A} = \dfrac{[MLT^{-2}]}{[L^2]} = [ML^{-1}T^{-2}]$
- Surface Tension (T) $= \dfrac{F}{l} = \dfrac{[MLT^{-2}]}{[L]} = [MT^{-2}]$
- Young's modulus (Y) $= \dfrac{F}{A} \times \dfrac{\ell}{\Delta\ell} = \dfrac{[MLT^{-2}] \times [L]}{[L^2] \times [L]} = [ML^{-1}T^{-2}]$
- Bulk's modulus $= \dfrac{P}{\Delta V / V} = \dfrac{[ML^{-1}T^{-2}]}{[L^3]/[L^3]} = [ML^{-1}T^{-2}]$
- Velocity Gradient $= \dfrac{\Delta v}{\Delta x} = \dfrac{[LT^{-1}]}{[L]} = [T^{-1}]$
- Latent heat(L) $= \dfrac{Q}{m} = \dfrac{[\cancel{M}L^2T^{-2}]}{[\cancel{M}]} = [M^0L^2T^{-2}]$
- Specific heat capacity(c) $= \dfrac{Q}{m\Delta\theta} = \dfrac{[\cancel{M}L^2T^{-2}]}{[\cancel{M}][\theta]} = [M^0L^2T^{-2}\theta^{-1}]$
- Charge (q) = It = [AT]
- Potential or Potential difference or EMF $= \dfrac{W}{q} = \dfrac{[ML^2T^{-2}]}{[AT]} = [ML^2T^{-3}A^{-1}]$

- Resistance (R) $= \frac{V}{I} = \frac{[ML^2T^{-3}A^{-1}]}{[A]} = [ML^2T^{-3}A^{-2}]$
- Self inductance(L) $= [M^1L^2T^{-2}A^{-2}]$
- Capacitance (C) $= \frac{Q}{V} = \frac{[AT]}{[ML^2T^{-3}A^{-1}]} = [M^{-1}L^{-2}T^4A^2]$
- Intensity of electric field (E) $= \frac{F}{q} = \frac{[MLT^{-2}]}{[AT]} = [MLT^{-3}A^{-1}]$
- Potential gradient $= \frac{\Delta V}{\Delta r} = \frac{[ML^2T^{-3}A^{-1}]}{[L]} = [MLT^{-3}A^{-1}]$
- Rate of decay (R) $= \frac{dN}{dt} = \frac{1}{[T]} = [T^{-1}]$
- Magnetic field intensity (B) $= \frac{F}{q \times v} = \frac{[MLT^{-2}]}{[AT] \times [LT^{-1}]} = [MT^{-2}A^{-1}]$

• Some Dimensionless Physical Quantities

Angle $= \frac{arc}{radius} = \frac{[L]}{[L]}$	Dimensionless	Unit = Radian
Refractive Index $(\mu) = \frac{c}{v} = \frac{[LT^{-1}]}{[LT^{-1}]}$	Dimensionless	Unitless
Coefficient of friction $(\mu) = \frac{f_L}{N} = \frac{[MLT^{-2}]}{[MLT^{-2}]}$	Dimensionless	Unitless
Relative density (ρ_r) or Specific gravity $= \frac{\rho}{\rho_{water}} = \frac{[ML^{-3}]}{[ML^{-3}]}$	Dimensionless	Unitless
Strain $= \frac{\Delta l}{l} = \frac{[L]}{[L]}$	Dimensionless	Unitless
Reynold's Number (R_N)	Dimensionless	Unitless
Dielectric Constant (K) $= \frac{E_0}{E_{net}}$	Dimensionless	Unitless
Sound level (SL) $= 10\log_{10}\frac{I}{I_0}$	Dimensionless	Unit=Decibel

Points to Remember (PTR)

- All unitless quantities are dimensionless but all dimensionless quantities are not necessarily unitless.
- All Trigonometric functions, Logarithmic functions and exponential functions are dimensionless in any formula or equation.

Example : $\sin\frac{x}{y}$ $\log\frac{xy}{z}$ $e^{xt/z}$

$\frac{x}{y}$ =Dimensionless $\frac{xy}{z}$ =Dimensionless xt/z= Dimensionless

DIMENSIONAL FORMULA OF SOME CONSTANT

- Universal gravitational constant (G):

$$F = G\frac{m_1 m_2}{r^2} \qquad G = \frac{Fr^2}{m_1 m_2} = \frac{[\cancel{M}LT^{-2}]\times[L^2]}{[\cancel{M}]\times[M]} = [M^{-1}L^3T^{-2}]$$

- Force constant or spring constant (k):

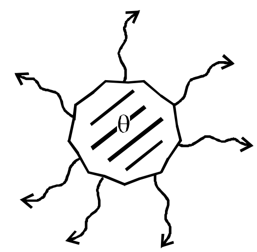
$$F = -kx \qquad k = \frac{F}{x} = \frac{MLT^{-2}}{L} = MT^{-2}$$

- Stefan's constant (σ):

Radiant power (P) $= \sigma\varepsilon A\theta^4$ — A= exposed surface area

$[M\cancel{L^2}T^{-3}] = \sigma\times[\cancel{L^2}]\times[K^4]$ — ε = Emissivity of body

$\sigma = [MT^{-3}K^{-4}]$ — θ = Absolute temperature



- Boltzmann constant (k):

Average K.E. of gaseous molecule $= \frac{3}{2}k\theta \qquad [ML^2T^{-2}] = k\times[\theta] \qquad k = [ML^2T^{-2}\theta^{-1}]$

- Planck's Constant (h):

Energy of photon $E = h\nu \qquad h = \frac{E}{\nu} = \frac{[ML^2T^{-2}]}{[T^{-1}]} \qquad h = [ML^2T^{-1}]$

- Wein's Constant (b):

$\lambda_m T = b \qquad b = [L\theta] \qquad \lambda_m$ = wavelength T=Temperature

- Faraday's constant (F): Charge of 1 mole electron

F =Q= [AT]

- Permittivity (ε_0):

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2} \qquad \varepsilon_0 = \frac{q_1q_2}{4\pi Fr^2} = \frac{[AT]\times[AT]}{[MLT^{-2}]\times[L^2]} \qquad \varepsilon_0 = [M^{-1}L^{-3}A^2T^4]$$

- Magnetic Permeability (μ_0):

$$B = \frac{\mu_0}{4\pi}.\frac{Idl\sin\theta}{r^2} \qquad \mu_0 = \frac{4\pi Br^2}{Idl\sin\theta} = \frac{[MT^{-2}A^{-1}][L^{\cancel{2}}]}{[A][\cancel{L}]} \qquad \mu_0 = [MLT^{-2}A^{-2}]$$

- Coefficient of Viscosity (η):

$$F = \eta\times A\times\frac{\Delta v}{\Delta x} \qquad \eta = \frac{F}{A\times\frac{\Delta v}{\Delta x}} = \frac{[M\cancel{L}T^{-2}]}{[L^{\cancel{2}}][T^{-1}]} \qquad \eta = [ML^{-1}T^{-1}]$$

- Hubble constant (H):

Relative velocity of galaxies $v \propto r$ $v = Hr$ $H = \frac{v}{r} = \frac{[LT^{-1}]}{[L]}$ $H = [T^{-1}]$

• DIMENSIONAL FORMULA OF SOME SPECIAL TERMS

- Dimensional formula of $\frac{1}{\sqrt{\mu_0\varepsilon_0}}$ or $\frac{E}{B}$:

The speed of E.M wave is given as $v = c = \frac{1}{\sqrt{\mu_0\varepsilon_0}} = \frac{E}{B}$

Dimensional formula of $\frac{1}{\sqrt{\mu_0\varepsilon_0}} = (\mu_0\varepsilon_0)^{-1/2}$ or $\frac{E}{B} = [LT^{-1}]$

- Dimensional formula of QV, CV^2 & $\frac{Q^2}{C}$:

Q = Charge C = Capacitance V = Potential difference

The energy stored in capacitor is given as $U = \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C}$

$\therefore$ Dimensional formula of $QV = CV^2 = \frac{Q^2}{C} = [ML^2T^{-2}]$

- Dimensional formula of CR, $\frac{L}{R}$ and $\sqrt{LC}$:

C = Capacitance R = Resistance L = Self Inductance

$q = q_0\left(1 - e^{-\frac{t}{CR}}\right)$ $\frac{t}{CR}$ = dimensionless D.F. of CR = D.F. of t CR = [T]

$I = I_0\left(1 - e^{-\frac{t}{L/R}}\right)$ $\frac{t}{L/R}$ = dimensionless D.F. of $\frac{L}{R}$ = D.F .of t $\frac{L}{R} = [T]$

Resonance frequency $\nu = \frac{1}{2\pi\sqrt{LC}}$ $\sqrt{LC} = \frac{1}{2\pi\nu} = \frac{1}{T^{-1}}$ $\sqrt{LC} = [T]$

- Dimensional formula of $\frac{1}{2}\varepsilon_0E^2$ and $\frac{1}{2}\frac{B^2}{\mu_0}$:

$$\frac{1}{2}\varepsilon_0E^2 \ \& \ \frac{1}{2}\frac{B^2}{\mu_0} = \text{Energy density}(u) = \frac{U}{\text{Volume}} = \frac{[ML^2T^{-2}]}{[L^3]} = [ML^{-1}T^{-3}]$$

POINTS TO PONDER

- Following are the physicalquantities having same dimensional formula-

S. No.	Quantities	Dimensions
1.	Strain, Refractive index, Relative density, Angle, Solid angle, Phase, Distance gradient, Relative permeability, Relative permittivity, Angle of contact, Reynold's number, Coefficient of friction, Mechanical equivalent of heat, Electric susceptibility, etc.	$[M^0L^0T^0]$
2.	Mass and Inertia	$[M^1L^0T^0]$
3.	Momentum and Impulse	$[M^1L^1T^{-1}]$
4.	Thrust, Force, Weight, Tension, Energy gradient.	$[M^1L^1T^{-2}]$
5.	Pressure, Stress, Young's modulus, Bulk modulus, Shear modulus, Modulus of rigidity, Energy density.	$[M^1L^{-1}T^{-2}]$
6.	Angular momentum and Planck's constant (h)	$[M^1L^2T^{-1}]$
7.	Acceleration, g and gravitational field intensity	$[M^0L^1T^{-2}]$
8.	Surface tension, Free surface energy (energy per unit area), force gradient, Spring constant	$[M^1L^0T^{-2}]$
9.	Latent heat and gravitational potential	$[M^0L^2T^{-2}]$
10.	Thermal capacity, Boltzman constant, Entropy	$[M^1L^2T^{-2}K^{-1}]$
11.	Work, Torque, Internal energy, Potential energy, Kinetic energy, Moment of force, (q^2/C), (Li^2), (qV), (V^2C), (I^2Rt), $\frac{V^2}{R}t$ (Vit), (PV), (RT), (mL), $(mc\Delta T)$	$[M^1L^2T^{-2}]$
12.	Frequency, Angular frequency, Angular velocity, Velocity gradient, Radioactivity $\frac{R}{L}$, $\frac{1}{RC}$, $\frac{1}{\sqrt{LC}}$	$[M^0L^0T^{-1}]$
13.	$\left(\frac{l}{g}\right)^{1/2}$, $\left(\frac{m}{k}\right)^{1/2}$, $\left(\frac{R}{g}\right)^{1/2}$, $\left(\frac{L}{R}\right)$, (RC), $(\sqrt{LC})$, time	$[M^0L^0T^1]$
14.	(VI), (I^2R) (V^2/R), Power	$[M^1L^2T^{-3}]$

- The dimension of a fundamental quantity in other fundamental quantity is always zero as a fundamental quantity cannot be written in terms of other fundamental quantity.

1.6 APPLICATION OF DIMENSIONAL ANALYSIS

● APPLICATION 1 : TO CONVERT A PHYSICAL QUANTITY FROM ONE SYSTEM OF UNITS TO OTHER

- This is based on a fact that magnitude of a physical quantity remains same whatever system is used for measurement i.e. magnitude = numeric value (n) × unit (u) = constant or $n_1u_1 = n_2u_2$

So if a quantity is represented by $[M^aL^bT^c]$ then

$$n_2 = n_1\frac{u_1}{u_2} = n_1\left(\frac{M_1}{M_2}\right)^a\left(\frac{L_1}{L_2}\right)^b\left(\frac{T_1}{T_2}\right)^c$$

n_2 = numerical value in II system, n_1 = numerical value in I system

M_2 = unit of mass in II system, M_1 = unit of mass in I system

L_2 = unit of length in II system, L_1 = unit of length in I system,

T_2 = unit of time in II system, T_1 = unit of time in I system

Example: Convert density $5gm/cm^3$ into Kg/m^3?

$n_1 = 5 \quad n_2 = ? \quad$ Dimensions of density $= [ML^{-3}]$ a=1 b= - 3

$$n_2 = 5 \times \left[\frac{M_1}{M_2}\right]^1 \left[\frac{L_1}{L_2}\right]^{-3} = 5 \times \left[\frac{gm}{Kg}\right]\left[\frac{cm}{m}\right]^{-3} \qquad n_2 = 5 \times \left[\frac{10^{-3}kg}{kg}\right]\left[\frac{10^{-2}m}{m}\right]^{-3}$$

$$n_2 = 5 \times 10^{-3} \times 10^6 \qquad n_2 = 5 \times 10^3$$

• APPLICATION 2 : TO CHECK THE DIMENSIONAL CONSISTENCY OF EQUATION

- **PRINCIPLE OF HOMOGENEITY OF DIMENSIONS:** Only those quantities can be added or subtracted which have same dimensional formula, multiplication and division of physical quantities with different dimensional formula are allowed.(किसी valid equation में अगर दो quantities add या subtract की गयी है तो दोनो terms का Dimensional formula same होगा)

  If $A + B = y$ or $A - B = y$ then D.F. of A=D.F of B.=D.F of y
- If an equation fails this consistency test, it is proved wrong, but if it passes, it is not proved right. Thus, a dimensionally correct equation need not be actually an exact(correct) equation, but a dimensionally wrong(incorrect) or inconsistent equation must be wrong. **[NCERT]**

Example: Check the accuracy of the relation T = $2\pi\sqrt{\frac{L}{g}}$ for a simple pendulum using dimensional analysis.

Solution: The dimensions of LHS = The dimension of T = $[M^0 L^0 T^1]$

The dimensions of RHS = $\left(\frac{\text{dimensions of length}}{\text{dimensions of acceleration}}\right)^{1/2} = \left[\frac{L}{LT^{-2}}\right]^{1/2} = [T^2]^{1/2} = [T] = [M^0 L^0 T^1]$

Since, the dimensions are same on both the sides, the relation is correct dimensionally.

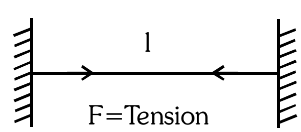
- The formula or equation is numerically correct or not, can only be determined by experiment.
- A formula which is dimensionally correct may or may not be numerically correct.For example, the formula of Time period of simple pendulum: $T = \sqrt{\frac{l}{g}}$ is dimensionally correct but numerically wrong.

• APPLICATION 3 : DEDUCING RELATION AMONG THE PHYSICAL QUANTITIES

- Using the same principle of homogeneity of dimensions new relations among physical quantities can be derived if the dependent quantities are known.

Example: The fundamental frequency(ν) of an stretched string depends on tension produced in it (F), length of string (l) and linear mass density (μ) then the formula for fundamental frequency is derived as

$$\mu = \frac{\text{mass}}{\text{length}} = \frac{[M]}{[L]} = [ML^{-1}]$$



$$\nu \propto F^a \ell^b \mu^c$$

$$\nu = K F^a \ell^b \mu^c$$

K= dimensionless constant

$[M^0L^0T^{-1}] = [MLT^{-2}]^a[L]^b[ML^{-1}]^c$

$[M^0L^0T^{-1}] = M^{a+c}L^{a+b-c}T^{-2a}$ on comparing LHS & RHS

$a+c = 0$ ....(1) $a+b-c = 0$ . ... (2) $-2a = 0-1$ .... (3)

$a = +\frac{1}{2}$ $c = -a = -\frac{1}{2}$ $+\frac{1}{2}+b+\frac{1}{2} = 0$ $b = -1$

therefore, $\nu = KF^{1/2}\ell^{-1}\mu^{-1/2}$ $\nu = \frac{K}{\ell}\sqrt{\frac{F}{\mu}}$ practically $K = \frac{1}{2}$ $\therefore \nu = \frac{1}{2\ell}\sqrt{\frac{F}{\mu}}$

Example : It is known that the time of revolution T of a satellite around the Earth depends on the universal gravitational constant G, the mass of the Earth M, and the radius of the circular orbit R. Obtain an expression for T using dimensional analysis.

Solution : We have $[T] \propto [G]^a [M]^b [R]^c$

$[M]^0 [L]^0 [T]^1 = [M]^{-a} [L]^{3a} [T]^{-2a} \times [M]^b \times [L]^c = [M]^{b-a} [L]^{c+3a} [T]^{-2a}$

Comparing the exponents

For [T] : $1 = -2a \Rightarrow a = -\frac{1}{2}$ **For [M]** : $0 = b - a \Rightarrow b = a = -\frac{1}{2}$

For [L] : $0 = c + 3a \Rightarrow c = -3a = \frac{3}{2}$

Putting the values we get $T \propto G^{-1/2} M^{-1/2} R^{3/2}$ $T \propto \sqrt{\frac{R^3}{GM}}$

The actual expression is $T = 2\pi\sqrt{\frac{R^3}{GM}}$

Example : Let in a new system universal gravitational constant (G) Planck's constant (h) and speed of light (c) are taken as fundamental quantities.Find the expression of mass in terms of G, h, c ?

Solution : $G = [M^{-1}L^3T^{-2}]$ $h = [ML^2T^{-1}]$ $c = [LT^{-1}]$

mass $= G^x h^y c^z$ $[M^1L^0T^0] = [M^{-1}L^3T^{-2}]^x[ML^2T^{-1}]^y[LT^{-1}]^z$

$M^1L^0T^0 = M^{-x+y}L^{3x+2y+z}T^{-2x-y-z}$

$-x+y = 1$ .... (1) $3x+2y+z = 0$ .... (2) $-2x-y-z = 0$ .... (3)

Eq. 1+Eq. 2+Eq.3 $2y = 1$ $y = \frac{1}{2}$ from eq. (1) $-x+\frac{1}{2} = 1$ $x = -\frac{1}{2}$

From eq. (2) $3\times\frac{-1}{2}+2\times\frac{1}{2}+Z = 0$ $Z = +\frac{1}{2}$

therefore, mass $= G^{-1/2}h^{1/2}c^{1/2}$ mass $= \sqrt{\frac{hc}{G}}$

NCERT EXAMPLE

Example 2.15 : Let us consider an equation $\frac{1}{2}mv^2 = mgh$ where *m* is the mass of the body, *v* its velocity, *g* is the acceleration due to gravity and *h* is the height. Check whether this equation is dimensionally correct.

Answer : The dimensions of LHS are $[M] [L T^{-1}]^2 = [M] [L^2 T^{-2}] = [M L^2 T^{-2}]$

The dimensions of RHS are $[M][L T^{-2}] [L] = [M][L^2 T^{-2}] = [M L^2 T^{-2}]$

The dimensions of LHS and RHS are the same and hence the equation is dimensionally correct.

Example 2.16 : The SI unit of energy is J = kg m^2 s^{-2}; that of speed *v* is m s^{-1} and of acceleration *a* is m s^{-2}. Which of the formulae for kinetic energy (*K*) given below can you rule out on the basis of dimensional arguments (*m* stands for the mass of the body) :

(a) $K = m^2 v^3$

(b) $K = \frac{1}{2}mv^2$

(c) K = ma

(d) $K = \frac{3}{16}mv^2$

(e) $K = \frac{1}{2}mv^2 + ma$

Answer : Since the kinetic energy *K* has the dimensions of $[M L^2 T^{-2}]$, formulas (a), (c) and (e) are ruled out. The correct formula for kinetic energy is given by (b).

Example 2.17 : Consider a simple pendulum, having a bob attached to a string, that oscillates under the action of the force of gravity. Suppose that the period of oscillation of the simple pendulum depends on its length (*l*), mass of the bob (*m*) and acceleration due to gravity (*g*). Derive the expression for its time period using method of dimensions.

Answer : The dependence of time period *T* on the quantities *l, g* and *m* as a product may be written as :

$$T = k\, l^x g^y m^z$$

where k is dimensionless constant and *x, y* and *z* are the exponents. By considering dimensions on both sides, we have

$$[L^0M^0T^1] = [L^1][L^1T^{-2}]^y[M^1]^z = L^{x+y}T^{-2y}M^z$$

On equating the dimensions on both sides, we have $x + y = 0$; $-2y = 1$; and $z = 0$ So that

$$x = \frac{1}{2}, y = -\frac{1}{2}, z = 0$$

Then, $T = k\, l^{1/2} g^{-1/2}$ or $T = k\sqrt{\frac{l}{g}}$

Note that value of constant k can not be obtained by the method of dimensions. Here it does not matter if some number multiplies the right side of this formula, because that does not affect its dimensions.

Actually, $k = 2\pi$ so that $k = 2\pi\sqrt{\frac{l}{g}}$

• LIMITATION OF APPLICATION OF DIMENSIONAL ANALYSIS

- In Mechanics the formula for a physical quantity depending on more than three physical quantities cannot be derived. It can only be checked.(क्योंकि equation 3 होगी और unknown 3 से ज्यादा, which cannot be solved)
- This method can be used only if the dependency is of multiplication type. The formulae containing exponential, trigonometrical and logarithmic functions can't be derived using this method. Formulae containing more than one term which are added or subtracted like $s = ut + at^2/2$ also can't be derived.
- The relation derived from this method gives no information about the dimensionless constants.
- If dimensions are given, physical quantity may not be unique as many physical quantities have the same dimensions.
- It gives no information whether a physical quantity is a scalar or a vector.

1.7 ERROR ANALYSIS

- Every measurement taken by any measuring instrument has some uncertainty, this uncertainty is called as **Error**.Therefore every calculated quantity based on measured values,also has an error. **[NCERT]**
- **Accuracy** of a measurement is a measure of how close the measured value is to the true value of the quantity.The accuracy in measurement may depend on several factors, including the limit or the resolution of the measuring instrument. **[NCERT]**
- **Precision** tells us to what resolutionor limit the quantity is measured. (कोई measurement में Decimal के जितना place बाद Digits होंगे वो उतना ही precise होगा). **[NCERT]**
- Error in a measurement is the difference between measured value and true or actual value.

$$\boxed{\text{Error} = \text{Measured value} - \text{True or actual value}}$$

- Errors may be of following two types –

(A) SYSTEMATIC OR CONTROLLABLE ERROR

- This error occurs in one direction, either positive or negative due to some known reasons like defect in instrument,imperfection in experimental technique or procedure,or Personal errors.
- Systematic errors can be minimised by improving experimental techniques, selecting better instruments and removing personal bias as far as possible. **[NCERT]**
- For a given set-up, these errors may be estimated to a certain extent and the necessary corrections may be applied to the readings. **[NCERT]**

(B) RANDOM OR UNCONTROLLABLE ERROR

- The random errors are those errors, which occur irregularly and hence are random with respect to sign and size. **[NCERT]**
- These can arise due to random and unpredictable fluctuations in experimental conditions (e.g. unpredict able fluctuations in temperature, voltage supply, mechanical vibrations of experimental set-ups, etc), personal (unbiased) errors by the observer taking readings, etc. **[NCERT]**

LEAST COUNT ERROR

- The smallest value which can be measured by the measuring instrument, is called as **least count**.The least count of an instrument is inversely proportional to the precision of the instrument.
- Least count error is the error associated with the resolution of instrument.
- Least count error is associated with both Systematic error and Random error.
- Least count error can be minimised by using higher precision instruments, improving experimental techniques,etc.
- Repeating the observations several times and taking the arithmetic mean of all the observations, the mean value would be very close to the true value of the measured quantity. **[NCERT]**

1.8 TERMS RELATED WITH ERROR

Let X_1, X_2, X_3 .... X_n are the readings or measured values taken in an experiment.

- **True Value OR Actual Value:** It is the mean or average of reading taken :

$$\overline{X} = \frac{X_1 + X_2 + X_3 + \dots\dots\dots + X_n}{n}$$

- **Absolute Error:** It is the error in individual readings with respect to true value and it is found by subtracting true value from measured value.

$\Delta X_1 = X_1 - \overline{X}$ $\Delta X_2 = X_2 - \overline{X}$ $\Delta X_3 = X_3 - \overline{X}$

.... $\Delta X_n = X_n - \overline{X}$ **(Absolute error may be positive or negative)**

- **Mean Absolute Error:** It is the possible error in true value and it is the average of absolute errors. (Only magnitude of absolute errors are taken)

$$\overline{\Delta X} = \frac{|\Delta X_1| + |\Delta X_2| + ...|\Delta X_n|}{n}$$

- **Relative or Fractional Error:** It is the mean absolute error with respect to true or actual value

$$\text{Relative or Fractional Error} = \frac{\overline{\Delta X}}{\overline{X}} \quad \text{Percentage Error} = \frac{\overline{\Delta X}}{\overline{X}} \times 100\%$$

- A measured physical quantity with error is written as $\left(\overline{X} \pm \overline{\Delta X}\right)$.
- Relative error and percentage error are unitless as well as dimensionless.
- अगर numericals में error ना दिया हो तो maximum possible error, instruments के least count के बराबर होगा।

Example : Following observations were taken with a vernier callipers while measuring the length of a cylinder.

3.29 cm, 3.28 cm, 3.29 cm, 3.31 cm,
3.28 cm, 3.27 cm, 3.29 cm, 3.30 cm

then find

(a) Most accurate length of the cylinder. (b) Absolute error in each observation.
(c) Mean absolute error (d) Relative error (e) Percentage error

Express the result in terms of absolute error and percentage error.

Solution:(a) Most accurate length of the cylinder will be the mean length $\left(\overline{\ell}\right)$

$$\overline{\ell} = \frac{3.29 + 3.28 + 3.29 + 3.31 + 3.28 + 3.27 + 3.29 + 3.30}{8} = 3.28875 \text{ cm} \qquad \text{or } \overline{\ell} = 3.29 \text{ cm}$$

(b) Absolute error in the first reading = 3.29 – 3.29 = 0.00 cm
Absolute error in the second reading = 3.28 – 3.29 = – 0.01 cm
Absolute error in the third reading = 3.29 – 3.29 = 0.00 cm
Absolute error in the forth reading = 3.31 – 3.29 = 0.02 cm
Absolute error in the fifth reading = 3.28 – 3.29 = – 0.01 cm
Absolute error in the sixth reading = 3.27 – 3.29 = – 0.02 cm
Absolute error in the seventh reading = 3.29 – 3.29 = 0.00 cm
Absolute error in the last reading = 3.30 – 3.29 = 0.01 cm

(c) Mean absolute error $= \overline{\Delta\ell} = \dfrac{0.00 + 0.01 + 0.00 + 0.02 + 0.01 + 0.02 + 0.00 + 0.01}{8} = 0.01 \text{ cm}$

(d) Relative error in length $= \dfrac{\overline{\Delta\ell}}{\overline{\ell}} = \dfrac{0.01}{3.29} = 0.0030395 = 0.003$

(e) Percentage error $= \dfrac{\overline{\Delta\ell}}{\overline{\ell}} \times 100 = 0.003 \times 100 = 0.3\%$

So length $l = 3.29 \text{ cm} \pm 0.01 \text{ cm}$ (in terms of absolute error)

or $l = 3.29 \text{ cm} \pm 0.30\%$ (in terms percentage error)

1.9 COMBINATION OF ERRORS

Let Δa, Δb and Δy are the errors in quantities a, b and y respectively

• IN ADDITION

Main formula : $\boxed{y = a + b}$

With Errors $y \pm \Delta y = (a \pm \Delta a) + (b \pm \Delta b)$ $\quad y \pm \Delta y = (a + b) \pm \Delta a \pm \Delta b$

$y \pm \Delta y = y \pm \Delta a \pm \Delta b$ $\quad \pm\Delta y = \pm \Delta a \pm \Delta b$

Maximum possible mean absolute error in y: $\boxed{\Delta y = \Delta a + \Delta b}$ **ERROR FORMULA**

• IN SUBTRACTION

Main formula : $\boxed{y = a - b}$

With error $y \pm \Delta y = (a \pm \Delta a) - (b \pm \Delta b)$ $\quad y \pm \Delta y = (a - b) \pm \Delta a \mp \Delta b$

$y \pm \Delta y = y \pm \Delta a \mp \Delta b$ $\quad \pm\Delta y = \pm\Delta a \mp \Delta b$

Maximum possible mean absolute error in y $\boxed{\Delta y = \Delta a + \Delta b}$ **ERROR FORMULA**

Example: The initial and final temperatures of water as recorded by an observer are $(40.6 \pm 0.2)^0$C and $(78.3 \pm 0.3)^0$C. Calculate the rise in temperature with proper error limits.

Solution: Given $\theta_1 = (40.6 \pm 0.2)^0C$ and $\theta_2 = (78.3 \pm 0.3)^0C$

Rise in temp. $\theta = \theta_2 - \theta_1 = 78.3 - 40.6 = 37.7^0C.$

$\Delta\theta = \pm(\Delta\theta_1 + \Delta\theta_2) = \pm(0.2 + 0.3) = \pm 0.5^0C$

$\therefore$ Rise in temperature $= (37.7 \pm 0.5)^0C$

❍ **Hence the rule : When two quantities are added or subtracted, the absolute error in the final result is the sum of the absolute errors in the individual quantities. [NCERT]**

• IN MULTIPLICATION

Main formula : $\boxed{y = a \times b}$

With error $y \pm \Delta y = (a \pm \Delta a) \times (b \pm \Delta b)$

$$y\left[1 \pm \frac{\Delta y}{y}\right] = a\left[1 \pm \frac{\Delta a}{a}\right] \times b\left[1 \pm \frac{\Delta b}{b}\right] \qquad y\left[1 \pm \frac{\Delta y}{y}\right] = (a \times b) \times \left[1 \pm \frac{\Delta a}{a}\right] \times \left[1 \pm \frac{\Delta b}{b}\right]$$

$$\cancel{1} \pm \frac{\Delta y}{y} = \cancel{1} \pm \frac{\Delta b}{b} \pm \frac{\Delta a}{a} + \frac{\Delta a \Delta b}{ab} \text{ (negligible)} \qquad \pm\frac{\Delta y}{y} = \pm\frac{\Delta a}{a} + \frac{\Delta b}{b}$$

Maximum possible fractional error $\boxed{\frac{\Delta y}{y} = \frac{\Delta a}{a} + \frac{\Delta b}{b}}$ **ERROR FORMULA**

$$\frac{\Delta y}{y} \times 100 = \frac{\Delta a}{a} \times 100 + \frac{\Delta b}{b} \times 100$$ **% Error in y = % Error in a + % Error in b**

Example: The length and breadth of a rectangle are (5.7 ± 0.1) cm and (3.4 ± 0.2) cm. Calculate area of the rectangle with error limits?

Solution: Given $\ell = (5.7 \pm 0.1)$ cm and $b = (3.4 \pm 0.2)$ cm

Area $A = \ell \times b = 5.7 \times 3.4 = 19.38$ cm^2

$$\frac{\Delta A}{A} = \pm\left(\frac{\Delta \ell}{\ell} + \frac{\Delta b}{b}\right) = \pm\left(\frac{0.1}{5.7} + \frac{0.2}{3.4}\right) = \pm\left(\frac{0.34 + 1.14}{5.7 \times 3.4}\right) = \pm\frac{1.48}{19.38}$$

or $\Delta A = \pm\frac{1.48}{19.38}\times A = \pm\frac{1.48}{19.38}\times 19.8 = \pm 1.48 \therefore$ Area = (19.38 ± 1.48) sq. cm

• IN DIVISION

Main formula : $\boxed{y = \frac{a}{b}}$

$$y \pm \Delta y = \frac{a \pm \Delta a}{b \pm \Delta b} \qquad y\left[1 \pm \frac{\Delta y}{y}\right] = \frac{a\left[1 \pm \frac{\Delta a}{a}\right]}{b\left[1 \pm \frac{\Delta b}{b}\right]} \qquad 1 \pm \frac{\Delta y}{y} = \left(1 \pm \frac{\Delta a}{a}\right)\left(1 \pm \frac{\Delta b}{b}\right)^{-1}$$

$$\cancel{1} \pm \frac{\Delta y}{y} = \cancel{1} \pm \frac{\Delta b}{b} \pm \frac{\Delta a}{a} - \frac{\Delta a \Delta b}{ab} \qquad \pm\frac{\Delta y}{y} = \pm\frac{\Delta a}{a} + \frac{\Delta b}{b}$$

Maximum possible fractional error $\boxed{\frac{\Delta y}{y} = \frac{\Delta a}{a} + \frac{\Delta b}{b}}$ **ERROR FORMULA**

$\frac{\Delta y}{y}\times 100 = \frac{\Delta a}{a}\times 100 + \frac{\Delta b}{b}\times 100$ **% Error in y = % Error in a + % Error in b**

- **Hence the rule : When two quantities are multiplied or divided, the relative error in the result is the sum of the relative errors in the multipliers. [NCERT]**

Example: A body travels uniformly a distance (13.8 ± 0.2) m in a time (4.0 ± 0.3) s. Calculate its velocity with error limits. What is the percentage error in velocity ?

Solution: Given distance s = (13.8 ± 0.2) m and time t = (4.0 ± 0.3) s

velocity $v = \frac{s}{t} = \frac{13.8}{4.0} = 3.45\ ms^{-1} = 3.5\ ms^{-1}$

$$\frac{\Delta v}{v} = \left(\frac{\Delta s}{s} + \frac{\Delta t}{t}\right) = \left(\frac{0.2}{13.8} + \frac{0.3}{4.0}\right) = \left(\frac{0.8 + 4.14}{13.8 \times 4.0}\right) = \frac{4.49}{13.8 \times 4.0} = \pm 0.0895$$

or $\Delta v = \pm 0.0895 \times 3.45 = \pm 0.3087 = \pm 0.31$

∴ velocity of body with error limits, is written as (3.5 ± 0.3) ms^{-1}

percentage error in velocity = $\frac{\Delta v}{v} \times 100 = \pm 0.0895 \times 100 = \pm 8.95\% = \pm 9\%$

NCERT EXAMPLE

Example 2.8 The temperatures of two bodies measured by a thermometer are
$t_1 = 20\ ^0C \pm 0.5\ ^0C$ and $t_2 = 50\ ^0C \pm 0.5\ ^0C$. Calculate the temperature difference and the error theirin.

Answer : $t' = t_2 - t_1 = (50\ ^0C \pm 0.5\ ^0C) - (20^0C \pm 0.5\ ^0C)$
$t' = 30\ ^0C \pm 1\ ^0C$

(b) Error of a product or a quotient

Suppose Z = AB and the measured values of A and B are $A \pm \Delta A$ and $B \pm \Delta B$. Then

$$Z \pm \Delta Z = (A \pm \Delta A)(B \pm \Delta B)$$

$$Z \pm \Delta Z = AB \pm B\ \Delta A + A\ \Delta B \pm \Delta A\ \Delta B$$

Dividing LHS by Z and RHS by AB we have, $1 \pm (\Delta Z/Z) = 1 \pm (\Delta A/A) \pm (\Delta B/B) \pm (\Delta A/A)(\Delta B/B)$.
Since ”A and ”B are small, we shall ignore their product.
Hence the maximum relative error

$\Delta Z/Z = (\Delta A/A) + (\Delta B/B)$. You can easily verify that this is true for division also.

Example 2.9 The resistance R = V/I where V = (100 ± 5)V and I = (10 ± 0.2)A. Find the percentage error in R.

Answer : The percentage error in V is 5% and in I it is 2%. The total error in R would therefore be 5% + 2% = 7%.

Example 2.10 Two resistors of resistances R_1 = 100 ±3 ohm and R_2 = 200 ± 4 ohm are connected (a) in series, (b) in parallel. Find the equivalent resistance of the (a) series combination, (b) parallel combination. Use for (a) the relation $R = R_1 + R_2$, and for (b) $\frac{1}{R'} = \frac{1}{R_1} + \frac{1}{R_2}$ and $\frac{\Delta R'}{R'^2} = \frac{\Delta R_1}{R_1^2} + \frac{\Delta R_2}{R_2^2}$

Answer (a) The equivalent resistance of series combination

$$R = R_1 + R_2 = (100 \pm 3)\ \text{ohm} + (200 \pm 4)\ \text{ohm} = 300 \pm 7\ \text{ohm}.$$

(b) The equivalent resistance of parallel combination

$$R' = \frac{R_1 R_2}{R_1 + R_2} = \frac{200}{3} = 66.7\,\text{ohm}$$

Then from $\frac{1}{R'} = \frac{1}{R_1} + \frac{1}{R_2}$

we get, $$\frac{\Delta R'}{R'^2} = \frac{\Delta R_1}{R_1^2} + \frac{\Delta R_2}{R_2^2}$$

$$\Delta R' = \left(R'^2\right)\frac{\Delta R_1}{R_1^2} + \left(R'^2\right)\frac{\Delta R_2}{R_2^2}$$

$$\Delta R' = \left(\frac{66.7}{100}\right)^2 3 + \left(\frac{66.7}{100}\right)^2 4 = 1.8$$

Then, $R' = 66.7 \pm 1.8$ ohm

(Here, ΔR is expresed as 1.8 instead of 2 to keep in confirmity with the rules of significant figures.)

Example 2.11 : Find the relative error in Z, if $Z = A^4 B^{1/3}/CD^{3/2}$.

Answer The relative error in *Z* is

$$\frac{\Delta Z}{Z} = 4\left(\frac{\Delta A}{A}\right) + \frac{1}{3}\left(\frac{\Delta B}{B}\right) + \left(\frac{\Delta C}{C}\right) + \frac{3}{2}\left(\frac{\Delta D}{D}\right)$$

Example 2.12 : The period of oscillation of a simple pendulum is $T = 2\pi\sqrt{\frac{L}{g}}$. Measured value of *L* is 20.0 cm known to 1 mm accuracy and time for 100 oscillations of the pendulum is found to be 90 s using a wrist watch of 1 s resolution. What is the accuracy in the determination of *g* ?

Answer : $$g = \frac{4\pi^2 L}{T^2}$$

Here, $T = \frac{t}{n}$ and $\Delta T = \frac{\Delta t}{n}$ therefore $\frac{\Delta T}{T} = \frac{\Delta t}{t}$

The errors in both *L* and *t* are the least count errors. Therefore,

$$\frac{\Delta g}{g} = \frac{\Delta L}{L} + 2\frac{\Delta T}{T} = \frac{0.1}{20.0} + 2\left(\frac{1}{90}\right) = 0.032$$

Thus, the percentage error in *g* is

$$100 \times \frac{\Delta g}{g} = 100 \times \frac{\Delta L}{L} + 2 \times 100 \times \frac{\Delta T}{T} = 3\%$$

• IN EQUATION OR RELATIONS HAVING POWERS

(a) **Main formula :** $y = a^n$

$$\boxed{\frac{\Delta y}{y} = n \times \frac{\Delta a}{a}}$$ **ERROR FORMULA**

% error in $y = n \times \%$ error in a

(b) **Main formula :** $y = a^m b^n$ and $y = \frac{a^m}{b^n}$ or $y = a^m \times b^{-n}$

$$\boxed{\frac{\Delta y}{y} = m \times \frac{\Delta a}{a} + n \times \frac{\Delta b}{b}}$$ **ERROR FORMULA**

% error in $y = m \times \%$ error in a + $n \times \%$ error in b

- The error formula remains unaffected whether the powers are +ve or –ve.

1.10 SIGNIFICANT FIGURE OR DIGITS

- The result of measurement should be reported in a way that indicates the precision of measurement.
- Normally, the reported result of measurement is a number that includes all digits in the number that are known reliably plus the first digit that is uncertain. The reliable digits plus the first uncertain digit are known as significant digits or significant figures. **[NCERT]**
- Significant figures in a measured value of a physical quantity tell the number of digits in which we have confidence. Larger the number of significant figures obtained in a measurement, greater is its accuracy and vice versa.
- Significant figures indicate, the precision of measurement which depends on the least count of the measuring instrument. **[NCERT]**
- A choice of change of different units does not change the number of significant digits or figures in a measurement. **[NCERT]**

- **Rules to find out the number of significant figures :**

I Rule : All the non-zero digits are significant e.g. 1984 has 4 SF.

II Rule: All the zeros between two non-zero digits are significant, no matter where the decimal point is, if at all.. e.g. 10806 has 5 SF.

III Rule : All the zeros to the left of first non-zero digit are not significant. e.g.00108 has 3 SF.

IV Rule: If the number is less than 1, zeros on the right of the decimal point but to the left of the first non-zero digit are not significant. e.g. 0.002308 has 4 SF.

V Rule: The trailing zeros (zeros to the right of the last non-zero digit) in a number with a decimal point are significant. e.g. 01.080 has 4 SF.

VI Rule: The trailing zeros in a number without a decimal point are not significant e.g. 010100 has 3 SF. But if the number comes from some actual measurement then the trailing zeros become significant.e.g. m = 100 kg has 3 SF.

VII Rule : When the number is expressed in exponential form, the exponential term does not affect the number of S.F. For example in $x = 12.3 = 1.23 \times 10^1 = .123 \times 10^2 = 0.0123 \times 10^3 = 123 \times 10^{-1}$ each term has 3 SF only.

- **Rules for arithmetical operations with significant figures :**

I Rule: In addition or subtraction the number of decimal places in the result should be equal to the number of decimal places of that term in the operation which contain lesser number of decimal places. e.g. 12.587 – 12.5 = 0.087 = 0.1 ($\because$ second term contain lesser i.e. one decimal place)

II Rule: In multiplication or division, the number of SF in the product or quotient is same as the smallest number of SF in any of the terms which are multiplied or divided. e.g. $5.0 \times 0.125 = 0.625 = 0.62$

POINTS TO PONDER

- To avoid the confusion regarding the trailing zeros of the numbers without the decimal point the best way is to report every measurement in *scientific notation* (in the power of 10). In this notation every number is expressed in the form $a \times 10^b$, where a is the base number between 1 and 10 and b is any positive or negative exponent of 10. The base number (a) is written in decimal form with the decimal after the first digit. While counting the number of SF only base number is considered (Rule VII).
- The change in the unit of measurement of a quantity does not affect the number of SF. For example in 2.308 cm = 23.08 mm = 0.02308 m = 23080 mm each term has 4 SF.

Example: Write down the number of significant figures in the following.

(a)	16 5	3SF (following rule I)
(b)	2.05	3 SF (following rules I & II)
(c)	34.000 m	5 SF (following rules I & V)
(d)	0.005	1 SF (following rules I & IV)
(e)	0.02340 N m^{-1}	4 SF (following rules I, IV & V)
(f)	26900	3 SF (see rule VI)
(g)	26900 kg	5 SF (see rule VI)

NCERT EXAMPLE

Example 2.13 : Each side of a cube is measured to be 7.203 m. What are the total surface area and the volume of the cube to appropriate significant figures?

Answer : The number of significant figures in the measured length is 4. The calculated area and the volume should therefore be rounded off to 4 significant figures.

Surface area of the cube = $6(7.203)^2\ m^2 = 311.299254\ m^2 = 311.3\ m^2$

Volume of the cube = $(7.203)^3\ m^3 = 373.714754\ m^3 = 373.7\ m^3$

Example 2.14 : 5.74 g of a substance occupies 1.2 cm^3. Express its density by keeping the significant figures in view.

Answer : There are 3 significant figures in the measured mass whereas there are only 2 significant figures in the measured volume.

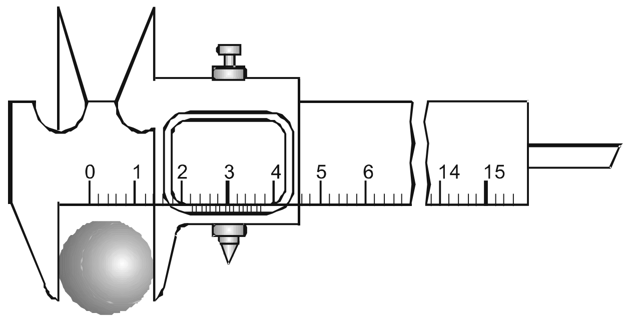
Hence the density should be expressed to only 2 significant figures.

$$\text{Density } 7 = \frac{5.74}{1.2}\,g\,cm^{-3} = 4.8\,g\,cm^{-3}$$

1.11 MEASURING INSTRUMENT

(1) VERNIER CALLIPER

- It is a device used to measure length, diameter and depth.



- **Least Count of Vernier Calliper :** Minimum value that can be measured by vernier calliper

MSD = Main scale Division (Main Scale पर एक division की value)

VSD = Vernier Scale Division (Vernier Scale पर एक Division की value)

Let n VSD are of same value as that of m MSD

$$n \text{ VSD} = m \text{ MSD} \qquad 1\text{VSD} = \frac{m}{n} \times 1 \text{ MSD}$$

Least Count or Vernier calliper is given as :- $= 1\text{MSD} - 1\text{VSD} = 1\text{MSD} - \frac{m}{n}\text{MSD}$

$$\boxed{\text{Least count} = 1\text{MSD} \times \left[1 - \frac{m}{n}\right]}$$

Usually n VSD=(n – 1) MSD ; (Example : 10 VSD=9 MSD)

$$\therefore \text{ L.C.} = \text{MSD}\left[1 - \frac{n-1}{n}\right] \quad \boxed{\text{L.C.} = \frac{1\text{MSD}}{n}}$$

- **ZERO ERROR :**
  - जब Vernier Calliper के दोनो jaws को मिला देंगे और Vernier Scale का zero Main Scale के zero से coincide कर जाऐ तो Calliper में कोई error नहीं होगा।
  - अगर Vernier Scale का zero mark, Main Scale के zero mark से right side पर होगा तो +ve zero error होगा और अगर left side पर होगा तो –ve zero error होगा।
  - Let N is a division of Vernier Scale which coincide with Main Scale when the jaws are touched together then : $\boxed{\text{zero error} = N \times \text{L.C.}}$

- **How to take measurement fromVernier Calliper :**

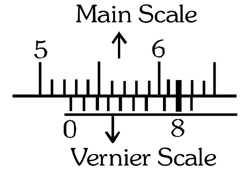
MSR = Main Scale Reading (Measurement लेते समय vernier scale के zero mark से पहले main scale की Reading)

VSR = Vernier Scale Reading (Coinciding division of Vernier Scale with Main Scale)

then, $\boxed{\text{Measured Value} = \text{MSR} + \text{VSR} \times \text{L.C.} - \text{zero error}}$ (put error with sign)

Example : MSR = 5.2 cm, VSR = 8, LC = 0.01 cm

Measured value $= 5.2 + 8 \times 0.01 = 5.28$ cm



❍

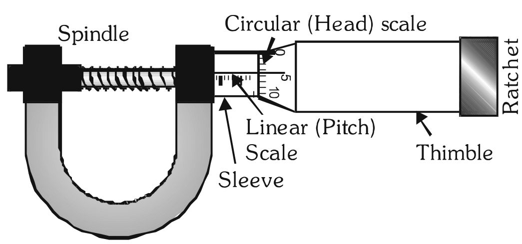
Example: The diameter of cylinder is measured using a Vernier Calliper with no zero error. It is found that zero of Vernier Scale lies between 5.10 cm and 5.15 of Main scale. The vernier scale has 50 divisions equivalent to 2.45 cm. If 24[th] division of vernier scale coincide with main scale then find the diameter of cylinder?

Solution: $MSD = 5.15 - 5.10$ $\qquad 1\,MSD = 0.05\,cm$ $\qquad 1VSD = \frac{2.45}{50}$

$LC = 1MSD - 1VSD$ $\qquad LC = 0.05 - \frac{2.45}{50}$ $\Rightarrow$ $LC = 0.001cm$

Diameter $= MSR + L.C \times V.S.R = 5.10 + 0.001 \times 24 = 5.10 + 0.024 = 5.124\,cm$

● **(2) MICROMETER SCREW GAUGE** : The screw gauge is an instrument used for measuring accurately the diameter of a thin wire or the thickness of sheet of metal.



● **Pitch of Screw Gauge:** Pitch is the linear displacement on main scale when Ratchet is rotated one turn.

$$\text{Pitch} = \frac{\text{Linear displacement on M.S.}}{\text{No of Rotations of Ratchet}}$$

● **Least Count of Screw Gauge :** Minimum value that can be measured by screw gauge.

$$\text{L.C.} = \frac{\text{Pitch (p)}}{\text{No. of divisions on circular scale(N)}} = \frac{p}{N}$$

● **Errors in Screw Gauge :**

(i) BACKLASH ERROR : Due to defect in Ratchet. Proper greasing करने से दूर होगा।

(ii) ZERO ERROR : जब Screw Gauge की rod और stud को मिला दिया जाएगा और circular scale का zero, main scale के reference line से coincide करेगा तो zero error नहीं होगा। अगर circular scale का zero reference line के नीचे होगा तो +ve zero error होगा और reference line के ऊपर होगा तो –ve zero error होगा।

Zero error = No. of divisions of circular scale below or above the reference line of main scale × L.C.

● **How to take measurement from Screw Gauge:**

MSR = Main Scale Reading (Reading लेते समय M.S की वो reading जो C.S से पहले दिखाई दे)

CSR = Circular Scale Reading (Circular Scale के Division की वो Value जो M.S. के Reference Line/ Base Line से coincide करें)

Measured value with error= MSR + LC $\times$ CSR – Zero error (put the sign of zero error)

Example: A student measured the diameter of small steel ball using screw gauge of L.C.= 0.001 cm. The MSR = 5mm and zero mark of circular scale coincides with 25 divisions of reference level. If the zero error is -0.004 cm then find the correct diameter of steel ball ?

Solution: Actual diameter $= 0.5 + 25 \times 0.001 + 0.004 \quad = 0.529\,\text{cm}$

● ADDITIONAL POINTS